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Gauge Solutions with Spherical Symmetry for the Massless Spin 2 Field

It is known that the system of Pauli-Firz equations for a massless field with spin 2 admits the existence of gauge solutions, they correspond to states that do not contribute to physically observable quantities such as the energy-momentum tensor. Such gauge solutions are determined by the known formulas through exact solutions of the equations for massless spin 1 field. In this paper, the problem is investigated in spherically symmetric case. Four independent solutions of the Duffin-Kemmer equation for a massless field with spin 1 in the spherical coordinate system are used. On this base, we found explicit form of four independent gauge solutions for spin 2 field. These solutions are presented in the form of linear combinations of Bessel functions with different indices, $J_p(\varepsilon r)$, $p = j-3/2, j-1/2, j+1/2, j+3/2, j+5/2$. *The established structure of gauge solutions can help in finding all solutions of the radial system of equations for a field with spin 2.*

Key words: *spin 2, Pauli – Fierz equation, massless field, spherical symmetry, spin 1 field, Duffin – Kemmer equation, gauge solutions, Bessel functions.*

КАЛИБРОВОЧНЫЕ РЕШЕНИЯ СО СФЕРИЧЕСКОЙ СИММЕТРИЕЙ ДЛЯ БЕЗМАССОВОГО ПОЛЯ СО СПИНОМ 2

Известно, что система уравнений Паули – Фирца для безмассового поля со спином 2 допускает существование калибровочных решений, им отвечают состояния, не дающие вклад в физически наблюдаемые величины типа тензор энергии-импульса. Эти калибровочные решения определяются согласно известным формулам через решения системы уравнений для безмассового поля со спином 1. В работе этот вопрос исследуется в сферически симметричном случае. Используются 4 независимые решения уравнения Даффина – Кеммера для безмассового поля со спином 1 в сферической системе координат. На этой основе получен явный вид четырех независимых калибровочных решений для поля со спином 2. Эти решения представляются в виде линейных комбинаций из функций Бесселя с различными индексами: $J_p(\varepsilon r)$, $p = j-3/2, j-1/2, j+1/2, j+3/2, j+5/2$. *Установленная структура калибровочных решений может помочь при нахождении всех решений радиальной системы уравнений для поля со спином 2.*

Ключевые слова: *спин 2, уравнение Паули – Фирца, безмассовое поле, сферическая симметрия, спин 1, уравнение Даффина – Кеммера, калибровочные решения, функции Бесселя.*

1. Introduction

After the investigation by Pauli and Fierz [1; 2], the theory of massive and massless fields is attracted much attention, for instance see [3–18; 21–25]. The most of the studies were performed in the framework of 2-nd order differential equations, as in [1; 2].

However it is known that many specific difficulties may be avoided if from the very beginning we start with the sysrtems of the first order equations. Apparently, the first systematic study of the theory of spin 2 fields within that formalism was performed by F. I. Fedorov [6]. It turns out that this description requires a field function with 30 independent components. This theory was re-discovered by Regee in [8].

It is known that in the theory of massless spin 1 particle (electromagnetic field) there exists the gauge symmetry, so all solutions of the gradient type do not contribute to physically observable quantities, like energy-momentum tensor.

Also it is known [1; 2] that the system of equations desribing the massless spin 2 field allows the existence of the gauge type solutions, which do not contribute to the physically observable quantities. They all are determined through an arbitrary vector fields $L_a(x)$ in accordance with the formulas [1; 2]

$$H(x) = e^{(c)\alpha}\partial_\alpha L_c(x) + e^{\alpha}_{(c);\alpha} L^c(x),$$

$$\Phi_{(ab)}(x) = -(\gamma_{[ca]b} + \gamma_{[cb]a})L^c(x) + [e^{\alpha}_{(a)}\partial_\alpha L_b(x) + e^{\alpha}_{(b)}\partial_\alpha L_a(x)] - \frac{1}{2} g_{ab}\Phi(x) ,$$

where $L^c(x)$ is a vector field; we assume the use of tetrad formalism [18, 19]. Below we will examine such solutions for spherically symmetric case.

2. Gauge solutions for massless spin 2 field

For scalar component referring to the spin 2 field, we have the following general expression (first we assume the use of Cartesian basis for components of vector field)

$$H(x) = \partial_t L_0(x) - (\partial_r + \frac{2}{r})L_3(x) - \frac{1}{r}[(\partial_\theta + \frac{\cos\theta}{\sin\theta})L_1(x) + \frac{\partial_\phi}{\sin\theta} L_2(x)]. \tag{1}$$

With the use of cyclic representation for vector components (see notations in [...])

$$L_0 = \bar{L}_0, \quad L_3 = \bar{L}_2, \quad L_1 = \frac{1}{\sqrt{2}}\left(-\bar{L}_1 + \bar{L}_3\right), \quad L_2 = -\frac{i}{\sqrt{2}}\left(\bar{L}_1 + \bar{L}_3\right), \tag{2}$$

we can rewrite relation (1) as follows

$$\bar{H}(x) = \partial_t \bar{L}_0 - (\partial_r + \frac{2}{r})\bar{L}_2 - \frac{1}{\sqrt{2}r}[(\partial_\theta + \frac{\cos\theta}{\sin\theta})\left(-\bar{L}_1 + \bar{L}_3\right) - i\frac{\partial_\phi}{\sin\theta}\left(\bar{L}_1 + \bar{L}_3\right)]. \tag{3}$$

Bearing in mind the substitution for 4-vector $\bar{L} = (\bar{L}_a)$:

$$\bar{L}(x) = e^{-i\epsilon t} \begin{vmatrix} \bar{L}_0(r)D_0 \\ \bar{L}_1(r)D_{-1} \\ \bar{L}_2(r)D_0 \\ \bar{L}_3(r)D_{+1} \end{vmatrix}, \tag{4}$$

we apply the technics of Wigner D-functions, for more details see in [19; 20], we transform relation (3) to the form

$$\bar{H} = e^{-i\epsilon t}\{-i\epsilon\bar{L}_0 - (\frac{d}{dr} + \frac{2}{r})\bar{L}_2 - \frac{1}{\sqrt{2}r}[-\bar{L}_1(\partial_\theta + \frac{-m+\cos\theta}{\sin\theta})D_{-1} + \bar{L}_3(\partial_\theta - \frac{-m-\cos\theta}{\sin\theta})D_{+1}]\}. \qquad (5)$$

With the use of the known recurrent formulas for Wigner functions [20.], we derive

$$\bar{h}(r) = -i\epsilon\bar{L}_0 - (\frac{d}{dr} + \frac{2}{r})\bar{L}_2 - \frac{a}{\sqrt{2}r}(\bar{L}_1 + \bar{L}_3). \qquad (6)$$

Now let us specify the gauge tensor Φ_{ab}, in Cartesian basis it is defined by the formulas [24; 25]:

$$\Phi_{00} = 2\partial_t L_0 - \frac{1}{2}h = f_0, \quad \Phi_{11} = \frac{2}{r}L_3 + \frac{2}{r}\partial_\theta L_1 + \frac{1}{2}h = f_1,$$
$$\Phi_{22} = 2\frac{\cos\theta}{r\sin\theta}L_1 + \frac{2}{r}L_3 + 2\frac{\partial_\phi}{r\sin\theta}L_2 + \frac{1}{2}h = f_2, \quad \Phi_{33} = 2\partial_r L_3 + \frac{1}{2}h = f_3,$$
$$\Phi_{01} = \partial_t L_1 + \frac{\partial_\theta}{r}L_0 = d_1, \quad \Phi_{02} = \partial_t L_2 + \frac{\partial_\phi}{r\sin\theta}L_0 = d_2, \quad \Phi_{03} = \partial_t L_3 + \partial_r L_0 = d_3, \qquad (7)$$
$$\Phi_{23} = -\frac{1}{r}L_2 + \frac{\partial_\phi}{r\sin\theta}L_3 + \partial_r L_2 = c_1, \Phi_{31} = -\frac{1}{r}L_1 + \partial_r L_1 + \frac{\partial_\theta}{r}L_3 = c_2,$$
$$\Phi_{12} = -\frac{\cos\theta}{r\sin\theta}L_2 + \frac{\partial_\theta}{r}L_2 + \frac{\partial_\phi}{r\sin\theta}L_1 = c_3.$$

Transition of the 10 components of the symmetric tensor Φ_{ab} to cyclic components is reached with the help of the following transformation (see in [24; 25]):

$$\begin{vmatrix} \bar{f}_1 \\ \bar{f}_2 \\ \bar{f}_3 \\ \bar{c}_1 \\ \bar{c}_2 \\ \bar{c}_3 \\ \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_3 \\ \bar{f}_0 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & -1 & . & . & . & -2i & . & . & . & . \\ . & . & 2 & . & . & . & . & . & . & . \\ 1 & -1 & . & . & . & 2i & . & . & . & . \\ . & . & . & i\sqrt{2} & \sqrt{2} & . & . & . & . & . \\ -1 & -1 & . & . & . & . & . & . & . & . \\ . & . & . & i\sqrt{2} & -\sqrt{2} & . & . & . & . & . \\ . & . & . & . & . & . & -\sqrt{2} & i\sqrt{2} & . & . \\ . & . & . & . & . & . & . & . & 2 & . \\ . & . & . & . & . & . & \sqrt{2} & i\sqrt{2} & . & . \\ . & . & . & . & . & . & . & . & . & 2 \end{vmatrix} \times$$

$$
\times\left|\begin{array}{c}
\frac{2}{r}L_3+\frac{2}{r}\partial_\theta L_1+\frac{1}{2}h=f_1\\
2\frac{\cos\theta}{r\sin\theta}L_1+\frac{2}{r}L_3+2\frac{\partial_\phi}{r\sin\theta}L_2+\frac{1}{2}h=f_2\\
2\partial_r L_3+\frac{1}{2}h=f_3\\
-\frac{1}{r}L_2+\frac{\partial_\phi}{r\sin\theta}L_3+\partial_r L_2=c_1\\
-\frac{1}{r}L_1+\partial_r L_1+\frac{\partial_\theta}{r}L_3=c_2\\
-\frac{\cos\theta}{r\sin\theta}L_2+\frac{\partial_\theta}{r}L_2+\frac{\partial_\phi}{r\sin\theta}L_1=c_3\\
\partial_t L_1+\frac{\partial_\theta}{r}L_0=d_1\\
\partial_t L_2+\frac{\partial_\phi}{r\sin\theta}L_0=d_2\\
\partial_t L_3+\partial_r L_0=d_3\\
2\partial_t L_0-\frac{1}{2}h=f_0
\end{array}\right|
=\frac{1}{r}\left|\begin{array}{c}
(m-\cos(\theta))\csc(\theta)\left(L_1-iL_2\right)+\frac{\partial}{\partial\theta}L_1-i\frac{\partial}{\partial\theta}L_2\\
\frac{1}{2}r\left(\Phi+4\frac{\partial}{\partial r}L_3\right)\\
-(m+\cos(\theta))\csc(\theta)\left(L_1+iL_2\right)+\frac{\partial}{\partial\theta}L_1+i\frac{\partial}{\partial\theta}L_2\\
\frac{-L_1-iL_2-m\csc(\theta)L_3+\frac{\partial}{\partial\theta}L_3+r\frac{\partial}{\partial r}L_1+ir\frac{\partial}{\partial r}L_2}{\sqrt{2}}\\
-\frac{r\Phi}{2}-\cot(\theta)L_1-im\csc(\theta)L_2-2L_3-\frac{\partial}{\partial\theta}L_1\\
-\frac{-L_1+iL_2+m\csc(\theta)L_3+\frac{\partial}{\partial\theta}L_3+r\frac{\partial}{\partial r}L_1-ir\frac{\partial}{\partial r}L_2}{\sqrt{2}}\\
\frac{-m\csc(\theta)L_0+r\epsilon\left(iL_1+L_2\right)-\frac{\partial}{\partial\theta}L_0}{\sqrt{2}}\\
r\left(\frac{\partial}{\partial r}L_0-i\epsilon L_3\right)\\
\frac{-m\csc(\theta)L_0+r\epsilon\left(L_2-iL_1\right)+\frac{\partial}{\partial\theta}L_0}{\sqrt{2}}\\
-\frac{1}{2}r\left(\Phi+4i\epsilon L_0\right)
\end{array}\right|.
$$

Expressing the Cartesian components of the vector through the cyclic ones

$$L_0=\bar{L}_0(r)D_0,\quad L_3=\bar{L}_2(r)D_0,$$

$$L_1=\frac{1}{\sqrt{2}}\left(-\bar{L}_1(r)D_{-1}+\bar{L}_3(r)D_{+1}\right),\quad L_2=-\frac{i}{\sqrt{2}}\left(\bar{L}_1(r)D_{-1}+\bar{L}_3(r)D_{+1}\right),$$

we obtain (reminding the identity $h=\bar{h}$)

$$\bar{H}_2=\frac{1}{r}\times$$

$$\times\begin{vmatrix} -\sqrt{2}\bar{L}_1\left((m\csc(\theta)-\cot(\theta))D_{-1}+D_{-1}'\right) \\ \frac{1}{2}r\left(\bar{h}D_0+4D_0\bar{L}_2'\right) \\ -\sqrt{2}\bar{L}_3\left((m+\cos(\theta))\csc(\theta)D_{+1}-D_{+1}'\right) \\ D_{+1}\left(r\bar{L}_3'-\bar{L}_3\right)+\frac{\bar{L}_2\left(D_0'-m\csc(\theta)D_0\right)}{\sqrt{2}} \\ \frac{1}{2}\left(-r\bar{h}D_0-4\bar{L}_2D_0+\sqrt{2}\left(\bar{L}_1\left((\cos(\theta)-m)\csc(\theta)D_{-1}+D_{-1}'\right)-\bar{L}_3\left((m+\cos(\theta))\csc(\theta)D_{+1}+D_{+1}'\right)\right)\right) \\ D_{-1}\left(r\bar{L}_1'-\bar{L}_1\right)-\frac{\bar{L}_2\left(m\csc(\theta)D_0+D_0'\right)}{\sqrt{2}} \\ -ir\epsilon\bar{L}_1D_{-1}-\frac{\bar{L}_0\left(m\csc(\theta)D_0+D_0'\right)}{\sqrt{2}} \\ r\left(D_0\bar{L}_0'-i\epsilon\bar{L}_2D_0\right) \\ \frac{\bar{L}_0\left(D_0'-m\csc(\theta)D_0\right)}{\sqrt{2}}-ir\epsilon\bar{L}_3D_{+1} \\ -\frac{1}{2}r\left(\bar{h}D_0+4i\epsilon\bar{L}_0D_0\right) \end{vmatrix}.$$

Applying the recurrent formulas for Wigner functions, we find the following representation for the Cartesian components of the gauge symmetric tensor

$$\begin{vmatrix} \bar{f}_1 \\ \bar{f}_2 \\ \bar{f}_3 \\ \bar{c}_1 \\ \bar{c}_2 \\ \bar{c}_3 \\ \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_3 \\ \bar{f}_0 \end{vmatrix}=\frac{1}{r}\begin{vmatrix} -\sqrt{2}b\bar{L}_1D_{-2} \\ r\left(\frac{\bar{h}}{2}+2\bar{L}_2'\right)D_0 \\ -\sqrt{2}b\bar{L}_3D_{+2} \\ -\frac{1}{2}\left(\sqrt{2}a\bar{L}_2+2\bar{L}_3-2r\bar{L}_3'\right)D_{+1} \\ \frac{r}{2}\left(-\bar{h}-\frac{1}{r}\left(4\bar{L}_2+\sqrt{2}a(\bar{L}_1+\bar{L}_3)\right)\right)D_0 \\ -\frac{1}{2}\left(2\bar{L}_1+\sqrt{2}a\bar{L}_2-2r\bar{L}_1'\right)D_{-1} \\ -\frac{1}{2}\left(\sqrt{2}a\bar{L}_0+2i\bar{L}_1r\epsilon\right)D_{-1} \\ r\left(\bar{L}_0'-i\bar{L}_2\epsilon\right)D_0 \\ -\frac{1}{2}\left(\sqrt{2}a\bar{L}_0+2i\bar{L}_3r\epsilon\right)D_{+1} \\ r\left(-\frac{\bar{h}}{2}-2i\bar{L}_0\epsilon\right)D_0 \end{vmatrix}, \qquad (8)$$

from (8) follow expressions for separate radial components:

$$\bar{f}_1 = -\frac{\sqrt{2}b}{r}\bar{L}_1, \quad \bar{f}_2 = \frac{\bar{h}}{2} + 2\bar{L}_2', \quad \bar{f}_3 = -\frac{\sqrt{2}b}{r}\bar{L}_3,$$
$$\bar{c}_1 = -\frac{a}{\sqrt{2}r}\bar{L}_2 - \frac{1}{r}\bar{L}_3 + \bar{L}_3', \quad \bar{c}_2 = -\frac{1}{2}\bar{h} - \frac{2}{r}\bar{L}_2 - \frac{a}{\sqrt{2}r}(\bar{L}_1 + \bar{L}_3),$$
$$\bar{c}_3 = -\frac{a}{\sqrt{2}r}\bar{L}_2 - \frac{1}{r}\bar{L}_1 + \bar{L}_1', \quad \bar{d}_1 = -\frac{a}{\sqrt{2}r}\bar{L}_0 - i\epsilon\bar{L}_1, \tag{9}$$
$$\bar{d}_2 = \bar{L}_0' - i\epsilon\bar{L}_2, \quad \bar{d}_3 = -\frac{a}{\sqrt{2}r}\bar{L}_0 - i\epsilon\bar{L}_3, \quad \bar{f}_0 = -\frac{\bar{h}}{2} - 2i\epsilon\bar{L}_0.$$

Let us impose the parity restrictions on the gauge vector $\bar{L}_a$ (see in [19]):

$$\bar{L}(x) = \begin{vmatrix} \bar{L}_0(r)D_0 \\ \bar{L}_1(r)D_{-1} \\ \bar{L}_2(r)D_0 \\ \bar{L}_3(r)D_{+1} \end{vmatrix}, \begin{matrix} P = (-1)^{j+1}, & \bar{L}_0(r) = 0, \bar{L}_2(r) = 0, \bar{L}_3(r) = -\bar{L}_1(r); \\ P = (-1)^j, & \bar{L}_3(r) = +\bar{L}_1(r). \end{matrix} \tag{10}$$

At the parity $P = (-1)^{j+1}$, we have

$$\bar{h} = -i\epsilon\bar{L}_0 - (\frac{d}{dr} + \frac{2}{r})\bar{L}_2 - \frac{a}{\sqrt{2}r}(\bar{L}_1 + \bar{L}_3) \equiv 0,$$
$$\bar{f}_1 = -\frac{1}{r}\sqrt{2}b\bar{L}_1, \quad \bar{f}_2 = 0, \quad \bar{f}_3 = \frac{\sqrt{2}b}{r}\bar{L}_1, \quad \bar{c}_1 = \frac{1}{r}\bar{L}_1 - \bar{L}_1', \quad \bar{c}_2 = 0, \tag{11}$$
$$\bar{c}_3 = -\frac{1}{r}\bar{L}_1 + \bar{L}_1', \quad \bar{d}_1 = -i\epsilon\bar{L}_1, \quad \bar{d}_2 = 0, \quad \bar{d}_3 = i\epsilon\bar{L}_1, \quad \bar{f}_0 = 0;$$

at the parity $P = (-1)^j$, we have

$$\bar{h} = -i\epsilon\bar{L}_0 - (\frac{d}{dr} + \frac{2}{r})\bar{L}_2 - \frac{2a}{\sqrt{2}r}\bar{L}_1,$$
$$\bar{f}_1 = -\frac{\sqrt{2}b}{r}\bar{L}_1, \ \bar{f}_3 = -\frac{\sqrt{2}b}{r}\bar{L}_1, \ \bar{f}_2 = -\frac{i\epsilon}{2}\bar{L}_0 - \frac{1}{2}(\frac{d}{dr} + \frac{2}{r})\bar{L}_2 - \frac{a}{\sqrt{2}r}\bar{L}_1 + 2\bar{L}_2',$$
$$\bar{c}_1 = -\frac{a}{\sqrt{2}r}\bar{L}_2 - \frac{1}{r}\bar{L}_1 + \bar{L}_1', \quad \bar{c}_3 = -\frac{a}{\sqrt{2}r}\bar{L}_2 - \frac{1}{r}\bar{L}_1 + \bar{L}_1',$$
$$\bar{c}_2 = +\frac{i\epsilon}{2}\bar{L}_0 + \frac{1}{2}(\frac{d}{dr} - \frac{2}{r})\bar{L}_2 - \frac{a}{\sqrt{2}r}\bar{L}_1 - \frac{2}{r}\bar{L}_2, \tag{12}$$
$$\bar{d}_1 = -\frac{a}{r\sqrt{2}r}\bar{L}_0 - i\epsilon\bar{L}_1, \quad \bar{d}_3 = -\frac{a}{\sqrt{2}r}\bar{L}_0 - i\epsilon\bar{L}_1, \quad \bar{d}_2 = \bar{L}_0' - i\epsilon\bar{L}_2,$$
$$\bar{f}_0 = +\frac{i\epsilon}{2}\bar{L}_0 + \frac{1}{2}(\frac{d}{dr} + \frac{2}{r})\bar{L}_2 + \frac{a\sqrt{2}}{2r}\bar{L}_1 - 2i\epsilon\bar{L}_0.$$

3. Explicit form of the gauge solutions for massless spin 2 field

Alternatively, we may ignore the above parity restrictions on components of the vector field L_a, and apply the general form of correspondence $L_a(x) \Rightarrow \Phi(x)$, $\Phi_{ab}(x)$ according to the formulas (assuming the use of dimensionless variable $z = \epsilon r$)

$$h = -iL_0 - (\frac{d}{dz} + \frac{2}{z})L_2 - \frac{a}{\sqrt{2}z}(L_1 + L_3),$$

$$f_1 = -\frac{\sqrt{2}b}{z}L_1, \quad f_2 = \frac{h}{2} + 2\frac{d}{dz}L_2, \quad f_3 = -\frac{\sqrt{2}b}{z}L_3,$$

$$c_1 = -\frac{a}{\sqrt{2}z}L_2 - \frac{1}{z}L_3 + \frac{d}{dz}L_3, \quad c_2 = -\frac{1}{2}h - \frac{2}{z}L_2 - \frac{a}{\sqrt{2}z}(L_1 + L_3), \quad c_3 = -\frac{a}{\sqrt{2}z}L_2 - \frac{1}{z}L_1 + \frac{d}{dz}L_1, \; d_1 = -\frac{a}{\sqrt{2}z}L_0 -$$

Let us write expressions for 4 independent vector fields $L_a^{(i)}(x)$, they are solutions of the equation for massless spin 1 field (see in [26]):

$$L_0^{(1)} = 0, \quad L_2^{(1)} = 0, \quad L_1^{(1)} = -L_3^{(1)} = \frac{1}{z^{1/2}}J_{j+1/2};$$

$$L_2^{(2)} = -i\frac{\sqrt{2}j}{z^{1/2}}J_{j-1/2}, \quad L_3^{(2)} = i\frac{a}{z^{1/2}}J_{j-1/2} = L_1^{(2)}, \quad L_0^{(2)} = -\frac{\sqrt{2}j}{z^{1/2}}J_{j+1/2};$$

$$L_1^{(3)} = L_3^{(3)} = i\frac{\sqrt{j}}{z^{1/2}}J_{j+3/2}, \quad L_2^{(3)} = i\frac{\sqrt{2}\sqrt{j+1}}{z^{1/2}}J_{j+3/2}, \quad L_0^{(3)} = -\frac{\sqrt{2}\sqrt{j+1}}{z^{1/2}}J_{j+1/2};$$

$$L_1^{(4)} = L_3^{(4)} = -\frac{1}{\sqrt{z}}\frac{a}{2j+1}J_{j-1/2} - \frac{1}{\sqrt{z}}\frac{a}{2j+1}J_{j+3/2},$$

$$L_2^{(4)} = \sqrt{2}\frac{j}{2j+1}\frac{1}{\sqrt{z}}J_{j-1/2} - \sqrt{2}\frac{j+1}{2j+1}\frac{1}{\sqrt{z}}J_{j+3/2}, \quad L_0^{(4)} = -i\frac{\sqrt{2}}{z^{1/2}}J_{j+1/2}, \tag{14}$$

where $J_p(z)$ denote the Bessel functions. For each vector field $L^{(1)},...,L^{(4)}$, we can find corresponding 11 gauge components.

For $L^{(1)} \Rightarrow \Phi_1$,

$$h = 0, \quad f_0 = 0, \quad f_1 = -\frac{\sqrt{2}b(J_{j-\frac{1}{2}} + J_{j+\frac{3}{2}})}{(2j+1)\sqrt{z}}, \quad f_3 = \frac{\sqrt{2}b(J_{j-\frac{1}{2}} + J_{j+\frac{3}{2}})}{(2j+1)\sqrt{z}}, \quad f_2 = 0,$$

$$c_1 = \frac{(j+2)J_{j+\frac{3}{2}} - (j-1)J_{j-\frac{1}{2}}}{(2j+1)\sqrt{z}}, \quad c_3 = -\frac{(j+2)J_{j+\frac{3}{2}} - (j-1)J_{j-\frac{1}{2}}}{(2j+1)\sqrt{z}}, \quad c_2 = 0,$$

$$d_1 = -\frac{iJ_{j+\frac{1}{2}}}{\sqrt{z}}, \quad d_3 = \frac{iJ_{j+\frac{1}{2}}}{\sqrt{z}}, \quad d_2 = 0, \quad h = 0$$

or differently

$$\Phi_1 = \frac{1}{\sqrt{z}}[\begin{vmatrix} 0 \\ -\frac{\sqrt{2}b}{(2j+1)} \\ 0 \\ \frac{\sqrt{2}b}{(2j+1)} \\ -\frac{j-1}{(2j+1)} \\ 0 \\ \frac{j-1}{(2j+1)} \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix} J_{j-1/2} + \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -i \\ 0 \\ i \\ 0 \end{vmatrix} J_{j+1/2} + \begin{vmatrix} 0 \\ -\frac{\sqrt{2}b}{(2j+1)} \\ 0 \\ \frac{\sqrt{2}b}{(2j+1)} \\ \frac{j+2}{(2j+1)} \\ 0 \\ -\frac{j+2}{(2j+1)} \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix} J_{j+3/2}].$$

$L^{(2)} \Rightarrow \Phi_2,$

$$h = 0, \quad f_0 = \frac{i\left(a^2 + j(7j-5)\right)J_{j+\frac{1}{2}}}{\sqrt{2}(2j-1)\sqrt{z}},$$

$$f_1 = -\frac{i\sqrt{2}ab\left(J_{j-\frac{3}{2}} + J_{j+\frac{1}{2}}\right)}{(2j-1)\sqrt{z}}, \quad f_3 = -\frac{i\sqrt{2}ab\left(J_{j-\frac{3}{2}} + J_{j+\frac{1}{2}}\right)}{(2j-1)\sqrt{z}},$$

$$f_2 = -i\frac{\left(a^2 + j(3j-5)\right)J_{j-\frac{3}{2}} + \left(a^2 - j(5j+1)\right)J_{j+\frac{1}{2}}}{\sqrt{2}(2j-1)\sqrt{z}},$$

$$c_1 = \frac{ia\left(2(j-1)J_{j-\frac{3}{2}} - J_{j+\frac{1}{2}}\right)}{(2j-1)\sqrt{z}}, \quad c_3 = \frac{ia\left(2(j-1)J_{j-\frac{3}{2}} - J_{j+\frac{1}{2}}\right)}{(2j-1)\sqrt{z}}, \quad c_2 = -\frac{i\left(a^2 + (j-3)j\right)\left(J_{j-\frac{3}{2}} + J_{j+\frac{1}{2}}\right)}{\sqrt{2}(2j-1)\sqrt{z}},$$

$$d_1 = \frac{a\left((3j+1)J_{j-\frac{1}{2}} + jJ_{j+\frac{3}{2}}\right)}{(2j+1)\sqrt{z}}, \quad d_3 \to \frac{a\left((3j+1)J_{j-\frac{1}{2}} + jJ_{j+\frac{3}{2}}\right)}{(2j+1)\sqrt{z}}, \quad d_2 = \frac{\sqrt{2}j\left((j+1)J_{j+\frac{3}{2}} - (3j+1)J_{j-\frac{1}{2}}\right)}{(2j+1)\sqrt{z}},$$

that is

$$\Phi_2 = \frac{1}{\sqrt{z}}\left[\begin{vmatrix} 0 \\ -\frac{i\sqrt{2}ab}{(2j-1)} \\ -\frac{2i\sqrt{2}j(j-1)}{(2j-1)} \\ -\frac{i\sqrt{2}ab}{(2j-1)} \\ \frac{ai}{2j-1}2(j-1) \\ -\frac{i\sqrt{2}j(j-1)}{(2j-1)} \\ \frac{ai}{2j-1}2(j-1) \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix} J_{j-3/2} + \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{a(3j+1)}{(2j+1)} \\ -\frac{\sqrt{2}j(3j+1)}{(2j+1)} \\ \frac{a(3j+1}{(2j+1)} \\ 0 \end{vmatrix} J_{j-1/2} + \begin{vmatrix} 0 \\ -\frac{i\sqrt{2}ab}{(2j-1)} \\ \frac{2i\sqrt{2}j^2}{(2j-1)} \\ -\frac{i\sqrt{2}ab}{(2j-1)} \\ -\frac{ai}{2j-1} \\ -\frac{i\sqrt{2}j(j-1)}{(2j-1)} \\ -\frac{ai}{2j-1} \\ 0 \\ 0 \\ 0 \\ 2i\sqrt{2}j \end{vmatrix} J_{j+1/2} + \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{aj}{2j+1} \\ \frac{\sqrt{2}j(j+1)}{(2j+1)} \\ \frac{aj}{2j+1} \\ 0 \end{vmatrix} J_{j+3/2}\right].$$

$L^{(3)} \Rightarrow \Phi_3$,

$$h = 0, \quad f_0 = i2\sqrt{2}\sqrt{j+1}J_{j+1/2},$$

$$f_1 = -\frac{i\sqrt{2}b\sqrt{j}\left(J_{j+\frac{1}{2}} + J_{j+\frac{5}{2}}\right)}{(2j+3)\sqrt{z}}, \quad f_3 = -\frac{i\sqrt{2}b\sqrt{j}\left(J_{j+\frac{1}{2}} + J_{j+\frac{5}{2}}\right)}{(2j+3)\sqrt{z}},$$

$$f_2 = 2i\sqrt{2}\sqrt{j+1}\left(\frac{j+1}{2j+3}\frac{1}{\sqrt{z}}J_{j+1/2} - \frac{j+2}{2j+3}\frac{1}{\sqrt{z}}J_{j+5/2}\right),$$

$$c_1 = -\frac{i\sqrt{j}}{2j+3}\frac{1}{\sqrt{z}}J_{j+1/2} - \frac{2i\sqrt{j}(j+2)}{(2j+3)\sqrt{z}}J_{j+5/2}, \quad c_3 = -\frac{i\sqrt{j}}{2j+3}\frac{1}{\sqrt{z}}J_{j+1/2} - \frac{2i\sqrt{j}(j+2)}{(2j+3)\sqrt{z}}J_{j+5/2},$$

$$c_2 = -\frac{i\sqrt{2}\sqrt{j+1}(j+2)}{2j+3}\left(J_{j+\frac{1}{2}} + J_{j+\frac{5}{2}}\right),$$

$$d_1 = \frac{(j+1)\sqrt{j}}{(2j+1)\sqrt{z}}J_{j-1/2} + \frac{(3j+2)\sqrt{j}}{(2j+1)\sqrt{z}}J_{j+3/2}, \quad d_3 = \frac{(j+1)\sqrt{j}}{(2j+1)\sqrt{z}}J_{j-1/2} + \frac{(3j+2)\sqrt{j}}{(2j+1)\sqrt{z}}J_{j+3/2}$$

$$d_2 = -\frac{j\sqrt{2}\sqrt{j+1}}{(2j+1)\sqrt{z}}J_{j-\frac{1}{2}} + \frac{\sqrt{2}\sqrt{j+1}(3j+2)}{(2j+1)\sqrt{z}}J_{j+\frac{3}{2}},$$

that is

$$\Phi_3=\frac{1}{\sqrt{z}}[\left|\begin{matrix}0\\0\\0\\0\\0\\0\\0\\ \frac{(j+1)\sqrt{j}}{(2j+1)}\\ -\frac{\sqrt{2}\sqrt{j+1}j}{(2j+1)}\\ \frac{(j+1)\sqrt{j}}{(2j+1)}\\0\end{matrix}\right| J_{j-1/2}+\left|\begin{matrix}0\\ -\frac{i\sqrt{2}\sqrt{j}b}{(2j+3)}\\ 2i\sqrt{2}\sqrt{j+1}\frac{j+1}{2j+3}\\ -\frac{i\sqrt{2}\sqrt{j}b}{(2j+3)}\\ -\frac{i\sqrt{j}}{2j+3}\\ -\frac{i\sqrt{2}\sqrt{j+1}}{(2j+3)}\\ -\frac{i\sqrt{j}}{2j+3}\\0\\0\\0\\ 2i\sqrt{2}\sqrt{j+1}\end{matrix}\right| J_{j+1/2}+\left|\begin{matrix}0\\0\\0\\0\\0\\0\\0\\ \frac{(3j+2)\sqrt{j}}{(2j+1)}\\ \frac{\sqrt{2}\sqrt{j+1}(3j+2)}{(2j+1)}\\ \frac{(3j+2)\sqrt{j}}{(2j+1)}\\0\end{matrix}\right| J_{j+3/2}+\left|\begin{matrix}0\\ -\frac{i\sqrt{2}\sqrt{j}b}{(2j+3)}\\ -2i\sqrt{2}\sqrt{j+1}\frac{j+2}{2j+3}\\ -\frac{i\sqrt{2}\sqrt{j}b}{(2j+3)}\\ -\frac{2i\sqrt{j}(j+2)}{(2j+3)}\\ -\frac{i\sqrt{2}\sqrt{j+1}}{(2j+3)}(j+2)\\ -\frac{2i\sqrt{j}(j+2)}{(2j+3)}\\0\\0\\0\\0\end{matrix}\right| J_{j+5/2}].$$

For the case $L^{(4)}\Rightarrow\Phi_4$

$$h=0,\quad f_0=-2\sqrt{2}\frac{1}{\sqrt{z}}J_{j+1/2},$$

$$f_1=\frac{\sqrt{2}ab}{\sqrt{z}}[\frac{1}{(2j+1)(2j-1)}J_{j-3/2}+\frac{2}{(2j-1)(2j+3)}J_{j+1/2}+\frac{1}{(2j+1)(2j+3)}J_{j+5/2}]=+f_3,$$

$$f_3=\frac{\sqrt{2}ab}{\sqrt{z}}[\frac{1}{(2j+1)(2j-1)}J_{j-3/2}+\frac{2}{(2j-1)(2j+3)}J_{j+1/2}+\frac{1}{(2j+1)(2j+3)}J_{j+5/2}],$$

$$f_2=\frac{(4j^2-4j)}{\sqrt{2}(2j-1)(2j+1)\sqrt{z}}J_{j-\frac{3}{2}}+\frac{2(-4j^2-4j+2)}{\sqrt{2}(2j-1)(2j+3)\sqrt{z}}J_{j+\frac{1}{2}}+\frac{(4j^2+12j+8)}{\sqrt{2}(2j+1)(2j+3)\sqrt{z}}J_{j+\frac{5}{2}},$$

$$c_1=2a[\frac{-(2j^2+j-3)}{(2j-1)(2j+1)(2j+3)\sqrt{z}}J_{j-\frac{3}{2}}+\frac{1}{(2j-1)(2j+3)\sqrt{z}}J_{j+\frac{1}{2}}+\frac{(2j^2+3j-2)}{(2j-1)(2j+1)(2j+3)\sqrt{z}}J_{j+\frac{5}{2}}],$$

$$c_3=2a[\frac{-(2j^2+j-3)}{(2j-1)(2j+1)(2j+3)\sqrt{z}}J_{j-\frac{3}{2}}+\frac{1}{(2j-1)(2j+3)\sqrt{z}}J_{j+\frac{1}{2}}+\frac{(2j^2+3j-2)}{(2j-1)(2j+1)(2j+3)\sqrt{z}}J_{j+\frac{5}{2}}],$$

$$c_2=\frac{(2j^2-2j)}{\sqrt{2}(2j-1)(2j+1)\sqrt{z}}J_{j-\frac{3}{2}}+\frac{2(2j^2+2j-2)}{\sqrt{2}(2j-1)(2j+3)\sqrt{z}}J_{j+\frac{1}{2}}+\frac{\left(2j^2+6j+4\right)}{\sqrt{2}(2j+1)(2j+3)\sqrt{z}}J_{j+\frac{5}{2}}$$

$$d_1=\frac{2ia\left(J_{j-\frac{1}{2}}+J_{j+\frac{3}{2}}\right)}{(2j+1)\sqrt{z}},\quad d_3=\frac{2ia\left(J_{j-\frac{1}{2}}+J_{j+\frac{3}{2}}\right)}{(2j+1)\sqrt{z}},\quad d_2=-\frac{2i\sqrt{2}\left(jJ_{j-\frac{1}{2}}-(j+1)J_{j+\frac{3}{2}}\right)}{(2j+1)\sqrt{z}}.$$

That is

$$\Phi_4 = \frac{1}{\sqrt{z}}\left[\begin{vmatrix} 0 \\ -\frac{\sqrt{2}ab}{(2j+1)(2j-1)} \\ \frac{(4j^2-4j)}{\sqrt{2}(2j-1)(2j+1)} \\ \frac{\sqrt{2}ab}{(2j+1)(2j-1)} \\ -\frac{2a(2j^2+j-3)}{(2j-1)(2j+1)(2j+3)} \\ \frac{(2j^2-2j)}{\sqrt{2}(2j-1)(2j+1)} \\ -\frac{2a(2j^2+j-3)}{(2j-1)(2j+1)(2j+3)} \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix} J_{j-3/2} + \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{2ia}{(2j+1)} \\ -\frac{2i\sqrt{2}j}{(2j+1)} \\ \frac{2ia}{(2j+1)} \\ 0 \end{vmatrix} J_{j-1/2} + \begin{vmatrix} 0 \\ \frac{2\sqrt{2}ab}{(2j-1)(2j+3)} \\ \frac{2(-4j^2-4j+2)}{\sqrt{2}(2j-1)(2j+3)} \\ \frac{2\sqrt{2}ab}{(2j-1)(2j+3)} \\ \frac{2a(2j+1)}{(2j-1)(2j+1)(2j+3)} \\ \frac{2(2j^2+2j-2)}{\sqrt{2}(2j-1)(2j+3)} \\ \frac{2a(2j+1)}{(2j-1)(2j+1)(2j+3)} \\ 0 \\ 0 \\ 0 \\ -2\sqrt{2} \end{vmatrix} J_{j+1/2} +\right.$$

$$\left. + \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{2ia}{(2j+1)} \\ \frac{2i\sqrt{2}(j+1)}{(2j+1)} \\ \frac{2ia}{(2j+1)} \\ 0 \end{vmatrix} J_{j+3/2} + \begin{vmatrix} 0 \\ \frac{\sqrt{2}ab}{(2j+1)(2j+3)} \\ \frac{(4j^2+12j+8)}{\sqrt{2}(2j+1)(2j+3)} \\ \frac{\sqrt{2}ab}{(2j+1)(2j+3)} \\ \frac{2a(2j^2+3j-2)}{(2j-1)(2j+1)(2j+3)} \\ \frac{\left(2j^2+6j+4\right)}{\sqrt{2}(2j+1)(2j+3)} \\ \frac{2a(2j^2+3j-2)}{(2j-1)(2j+1)(2j+3)} \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix} J_{j+5/2}\right].$$

In the following, the multiplier $1/\sqrt{z}$ will be omitted for brevity.

4. Non-gauge nature of the simplest solution Ψ

Let us recall the structure of the simplest solution which was found in [20] for states with parity $\Pi = (-1)^{j+1}$:

$$\Psi = \begin{vmatrix} 0 \\ \frac{\sqrt{2}}{b}\frac{j+2}{2j+1} \\ 0 \\ -\frac{\sqrt{2}}{b}\frac{j+2}{r2j+1} \\ \frac{1}{2j+1} \\ 0 \\ -\frac{1}{2j+1} \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix} J_{j-1/2} + \begin{vmatrix} 0 \\ -\frac{\sqrt{2}}{b}\frac{j-1}{2j+1} \\ 0 \\ \frac{\sqrt{2}}{b}\frac{j-1}{2j+1} \\ \frac{1}{2j+1} \\ 0 \\ -\frac{1}{2j+1} \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix} J_{j+3/2} =$$

$$= 0\cdot J_{j-3/2} + VJ_{j-1/2} + 0\cdot J_{j+1/2} + WJ_{j+3/2} + 0\cdot J_{j+5/2}. \qquad (15)$$

Four gauge solutions have the following structure

$$\Phi^1 = 0\cdot J_{j-3/2} + Y_1 J_{j-1/2} + P_1 J_{j+1/2} + R_1 J_{j+3/2} + 0\cdot J_{j+5/2},$$
$$\Phi^2 = X_2 J_{j-3/2} + Y_2 J_{j-1/2} + P_2 J_{j+1/2} + R_2 J_{j+3/2} + 0\cdot J_{j+5/2},$$
$$\Phi^3 = 0\cdot J_{j-3/2} + Y_3 J_{j-1/2} + P_3 J_{j+1/2} + R_3 J_{j+3/2} + S_3 J_{j+5/2},$$
$$\Phi^4 = X_4 J_{j-3/2} + Y_4 J_{j-1/2} + P_4 J_{j+1/2} + R_4 J_{j+3/2} + S_4 J_{j+5/2}. \qquad (16)$$

Let us assume that the equality exists

$$\Psi = A\Phi^1 + B\Phi^2 + C\Phi^3 + D\Phi^4. \qquad (17)$$

We should equate the columns from the left and the right at the Bessel functions with coinciding indices; the results are

$$J_{j-3/2}, \quad D = iB(1+2j), \quad A, C \ \textit{are arbitrary};$$
$$J_{j-1/2}, \quad \textit{no solutions}; \qquad J_{j+3/2}, \quad \textit{no solutions};$$
$$J_{j+1/2}, \quad A=0, B=0, C=0, D=0;$$
$$J_{j+5/2}, \quad C=0, D=0, \quad A, B \ \textit{are arbitrary}.$$

Whence we conclude that the last system does not have any solutions. In other words, the simplest solutions Ψ for states with parity $\Pi = (-1)^{j+1}$ cannot be decomposed in the linear combinations of 4 gauge solutions $\Phi_1,...,\Phi_4$. Therefore, these solutions are physically observable (non-gauge).

Additionally, we may prove the linear independence of 4 gauge solutions. To this end we should examine the following equation

$$A\Phi_1 + B\Phi_2 + C\Phi_3 + D\Phi_4 = 0. \quad (18)$$

Taking into account the structure of gauge solutions, we derive the algebraic system

$$AY_1 + BY_2 + CY_3 + DY_4 = 0, \quad AP_1 + BP_2 + CP_3 + DP_4 = 0,$$
$$AR_1 + BR_2 + CR_3 + DR_4 = 0, \quad BX_2 + DX_4 = 0, \quad CS_3 + DS_4 = 0.$$

Taking into account the explicit expressions for all involved columns, we prove that the system has only the trivial solution

$$A = 0, \quad B = 0, \quad C = 0, \quad D = 0. \quad (19)$$

Therefore, the four gauge solutions are linearly independent.

5. Conclusions

Four independent solutions for spin 1 massless field, according to Pauli – Fierz theory, permit to find explicite form of 11-component gauge solutions for massless spin 2 field. These four solutions are expressed (up to multiplier $1/\sqrt{z}$) through Bessel functions with indices $p = j-3/2, j-1/2, j+1/2, j+3/2, j+5/2.$ Therefore we can assume that all possible solutions of the radial system for massless spin 2 field may be constructed within the same structure as linear combinations of Bessel functions.

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