УДК 539.12 DOI 10.63874/2218-0303-2025-1-29-47

Alina Ivashkevich
Junior Researcher of Fundamental Interaction and Astrophysics Center of B. I. Stepanov Institute of Physics of National Academy of Sciences of Belarus
Алина Валентиновна Ивашкевич
мл. научн. сотрудник центра фундаментальных взаимодействий и астрофизики Института физики имени Б. И. Степанова Национальной академии наук Беларуси
e-mail: ivashkevich.alina@yandex.by

Dirac – Kähler Particle in the Uniform Electric Field, Solutions with Cylindrical Symmetry

16-component system of equations describing the Dirac – Kähler particle in presence of the external electric field has been studied. This equation describes a multi-spin boson field equivalent to the scalar, pseudoscalar, vector, pseudovector,and anti-symmetric tensor. On the searched solutions, the operators of the energy and the third projection of the total angular momentum are diagonalized. After separating the variables in cylindrical coordinates, the system of sixteen first order equations in partial derivative with respect to coordinates (r, z) is derived. To resolve this system the method by Fedorov – Gronskiy is applied. Correspondingly, the complete wave function is decomposed into the sum of three projective constituents. Dependence of 16 variables on the polar coordinate is determined only through three basic functions $F_i(r)$, at this there arise differential constrains which permit to derive the system of 16 differential equations in the coordinate z. The three basic variables are found in terms of Bessel functions. The sysytem of equations in the variable z is solved exactly, as the result, four linearly independent solutions for the Dirac – Kähler particle in presence of the external uniform electric field are constructed.

Key words: *Dirac – Kähler particle, external electric field, cylindrical symmetry, projective operators, differential equations in partial derivatives, exact solutions.*

ЧАСТИЦА ДИРАКА – КЭЛЕРА В ОДНОРОДНОМ ЭЛЕКТРИЧЕСКОМ ПОЛЕ, РЕШЕНИЯ С ЦИЛИНДРИЧЕСКОЙ СИММЕТРИЕЙ

Исследуется 16-компонентная система уравнений, описывающая частицу Дирака – Кэлера в присутствии внешнего электрического поля. Эти уравнения описывают мультиспиновое бозонное поле, эквивалентное скаляру, псевдоскаляру, вектору, псевдовектору и антисимметричному тензору. На строящихся решениях диагонализируются операторы энергии и третьей проекции полного углового момента. После разделения переменных выведена система из 16-ти дифференциальных уравнений первого порядка в частных производных по координатам (r, z). Чтобы решить эту систему, используем метод Федорова – Гронского. Согласно этому методу, полная волновая функция раскладывается на сумму трех проективных составляющих. Зависимость 16 переменных от полярной координаты определяется только через три базисные функции $F_i(r)$. Накладываются дополнительные дифференциальные ограничения, которые позволяют преобразовать все уравнения в частных производных в систему обыкновенных дифференциальных уравнений по переменной z. Три основные переменные найдены в терминах функций Бесселя. Система уравнений по переменной z решается точно, в результате построены четыре линейно независимых решения уравнения для частицы Дирака – Кэлера в однородном электрическом поле.

Ключевые слова: *частица Дирака – Кэлера, внешнее электрическое поле, цилиндрическая симметрия, проективные операторы, уравнения в частных производных, точные решения.*

Introduction

As the Dirac equation was proposed [1], there appeared papers in which they tried to reproduce Dirac's results within the tensor representations of the Lorentz group [2; 3]. Later, and equivalent approach within the other mathematical techniques was proposed by Kähler [4], now mostly used the term term Dirac – Kähler field (also there are used other terms as well: Ivanenko – Landau field, or vector field of the general type). Scientific literature concerned with this field is enormous – see in [5–23].

Three most interesting points in connection of general covariant extension of the wave equation for this field are: in flat Minkowski space there exist tensor and spinor formulations of the theory; in the initial tensor form there are presented tensors with different intrinsic parities; there exist different views about physical interpretation of the object: whether it is a composite boson or a set of four fermions.

In Minkowski space and Cartesian coordinates, the Dirac – Kähler equation may be presented as a spinor equation for a 2-rank bispinor [7]:

$$[i\,\gamma^a \partial_a - m\,]U(x) = 0\,, \tag{1}$$

or as the set of tensor equations [7]:

$$\begin{aligned}
&\partial_l \Psi + m\Psi_l = 0\,, \qquad \partial_l \tilde{\Psi} + m\tilde{\Psi}_l = 0\,,\\
&\partial_l \Psi + \partial_a \Psi_{la} - m\Psi_l = 0\,,\\
&\partial_l \tilde{\Psi} - \frac{1}{2}\epsilon_l^{\ amn}\,\partial_a \Psi_{mn} - m\tilde{\Psi}_l = 0\,,\\
&\partial_m \Psi_n - \partial_n \Psi_m + \epsilon_{mn}^{\ \ ab}\,\partial_a \tilde{\Psi}_b - m\Psi_{mn} = 0\,.
\end{aligned} \tag{2}$$

These two description are related in the following way [7]:

$$U(x) = \left[-i\Psi + \gamma^l\,\Psi_l + i\sigma^{mn}\,\Psi_{mn} + \gamma^5\,\tilde{\Psi} + i\gamma^l\gamma^5\,\tilde{\Psi}_l\,\right]E^{-1}\,, \tag{3}$$

the quantity E is a bispinor metrical matrix

$$E = \begin{vmatrix} \epsilon_{\alpha\beta} & 0 \\ 0 & \epsilon^{\dot{\alpha}\dot{\beta}} \end{vmatrix} = \begin{vmatrix} i\sigma^2 & 0 \\ 0 & -i\sigma^2 \end{vmatrix}; \tag{4}$$

the inverse transformation is

$$\begin{gathered}
\Psi = -\frac{1}{4i}\,Sp\,[EU]\,, \quad \tilde{\Psi} = \frac{1}{4}\,Sp\,[E\gamma^5 U)]\,,\\
\Psi_l = \frac{1}{4}\,Sp\,[E\gamma_l U]\,, \tilde{\Psi}_l = \frac{1}{4i}\,Sp\,[E\gamma^5\gamma_l U]\,, \Psi_{mn} = -\frac{1}{2i}\,Sp\,[E\sigma_{mn} U]\,.
\end{gathered} \tag{5}$$

In this paper we will examine the problem for this particle in the external uniform electric field. We specify the main equation in cylindrical coordinates and then separate the variables. As a result, derive the system of 16 first order differential equations in partial derivatives with respect to the coordinates r and z.

To resolve this system, we apply the generalized method by Fedorov – Groskiy [24], the last is based on the use of projective operators constructed with the use of generator J^{12} for the wave function with the properties of 2nd rank bispinor. Correspondingly, we present the complete wave function into the sum of three projective constituents. Dependence of each projective constituent on the polar coordinate is determined trough one function. We introduce differential constariants which permit us to transform 16 equations in partiall derivatives to ordinartrt diddfereebtiola equations in the variable z. We study the introduced differential constraints for basic $F_1(r), F_2(r), F_3(r)$. They lead to 2nd order equations for basic three functions, solutions for them may be found in terms of the Bessel functions. The sysytem of equations in the variable z is solved exactly, as the result we have constructed four linearly independent solutions for the Dirac – Kähler particle in the uniform electric field.

1. Separating the variables, the method by Fedorov – Gronsliy

In the tetrad form , the above equation reads [7]

$$[i\gamma^{\alpha}(x)(D_{\alpha}+\Gamma_{\alpha}\otimes I+I\otimes\Gamma_{\alpha}(x)-M]U(x)=0,\quad D_{\alpha}=\partial_{\alpha}+ieA_{\alpha}. \tag{6}$$

In presence of the external uniform electric field, $A_0 = Ez$, it takes the form

$$[i\gamma^{0}(\frac{\partial}{\partial t}+iEz)+i\gamma^{1}\partial_{r}+\frac{\gamma^{2}}{r}(i\partial_{\phi}+(\sigma^{12}\otimes I+I\otimes\sigma^{12}))+i\gamma^{3}\partial_{z}-M]U(x)=0,$$

or differently (let $U(x)=\dfrac{\Phi(x)}{\sqrt{r}}$)

$$[i\gamma^{0}(\frac{\partial}{\partial t}+iEz)+i\gamma^{1}\partial_{r}+\frac{1}{r}\gamma^{2}\otimes\sigma^{12}+\frac{\gamma^{2}}{r}i\partial_{\phi}+i\gamma^{3}\partial_{z}-M]\Phi(x)=0. \tag{7}$$

Solutions are searched in the form [8]

$$V(t,r,\phi,z)=e^{-i\epsilon t}e^{im\phi}F(r),\quad F(r)=\begin{vmatrix} f_{11}(r,z) & f_{12}(r,z) & f_{13}(r,z) & f_{14}(r,z)\\ f_{21}(r,z) & f_{22}(r,z) & f_{23}(r,z) & f_{24}(r,z)\\ f_{31}(r,z) & f_{32}(r,z) & f_{33}(r,z) & f_{34}(r,z)\\ f_{41}(r,z) & f_{42}(r,z) & f_{43}(r,z) & f_{44}(r,z)\end{vmatrix}, \tag{8}$$

then the equation reads

$$[(\epsilon-Ez)\gamma^{0}+i\gamma^{1}\frac{\partial}{\partial r}+\frac{1}{r}\gamma^{2}\otimes\sigma^{12}-\frac{m}{r}\gamma^{2}+i\gamma^{3}\partial_{z}-M]F(x)=0. \tag{9}$$

Recall expressions for the Dirac matrices in spinor basis [7]

$$\gamma^{0}=\begin{vmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{vmatrix},\quad \gamma^{1}=\begin{vmatrix}0&0&0&-1\\0&0&-1&0\\0&1&0&0\\1&0&0&0\end{vmatrix},\quad \gamma^{2}=\begin{vmatrix}0&0&0&i\\0&0&-i&0\\0&-i&0&0\\i&0&0&0\end{vmatrix},$$

$$\gamma^{3}=\begin{vmatrix}0&0&-1&0\\0&0&0&1\\1&0&0&0\\0&-1&0&0\end{vmatrix},\quad \sigma^{12}=\begin{vmatrix}1/2&0&0&0\\0&-1/2&0&0\\0&0&1/2&0\\0&0&0&-1/2\end{vmatrix}.$$

Below we will apply the method by Fedorov – Gronskiy [24]. To this end, we present the complete wave function as the 16-dimensional column

$$F=\begin{vmatrix}F_1(r,z)\\F_2(r,z)\\F_3(r,z)\\F_4(r,z)\end{vmatrix},\quad F_1=\begin{vmatrix}f_{11}(r,z)\\f_{21}(r,z)\\f_{31}(r,z)\\f_{41}(r,z)\end{vmatrix}, F_2=\begin{vmatrix}f_{12}(r,z)\\f_{22}(r,z)\\f_{32}(r,z)\\f_{42}(r,z)\end{vmatrix}, F_3=\begin{vmatrix}f_{13}(r,z)\\f_{23}(r,z)\\f_{33}(r,z)\\f_{43}(r,z)\end{vmatrix}, F_4=\begin{vmatrix}f_{14}(r,z)\\f_{24}(r,z)\\f_{34}(r,z)\\f_{44}(r,z)\end{vmatrix}.$$

Correspondingly we should present the third projection of the spin $Y=S_3$ in 16-dimensional form.

$$
Y=\begin{vmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1
\end{vmatrix}, \tag{10}
$$

We readily verify that the minimal equation for this matrix has the form $Y^3 - Y = 0$; correspondingly, there exist three projective operators

$$
P_1 = \frac{1}{2}Y(Y+1), \quad P_2 = \frac{1}{2}Y(Y-1), \quad P_3 = 1 - Y^2, \tag{11}
$$

dependence of each constituent on the variable r should be determined through one function:

$$
\Psi_1 = \begin{vmatrix}
f_{11}(z)\\ 0\\ \left(f_{13}(z)+f_{31}(z)\right)/2\\ 0\\ 0\\ 0\\ 0\\ \left(f_{42}(z)-f_{24}(z)\right)/2\\ \left(f_{13}(z)+f_{31}(z)\right)/2\\ 0\\ f_{33}(z)\\ 0\\ 0\\ -\left(f_{42}(z)-f_{24}(z)\right)/2\\ 0\\ 0
\end{vmatrix} F_1(r), \Psi_2 = \begin{vmatrix}
0\\ 0\\ -\left(f_{13}(z)-f_{31}(z)\right)/2\\ 0\\ 0\\ f_{22}(z)\\ 0\\ \left(f_{42}(z)+f_{24}(z)\right)/2\\ \left(f_{13}(z)-f_{31}(z)\right)/2\\ 0\\ 0\\ 0\\ 0\\ \left(f_{42}(z)+f_{24}(z)\right)/2\\ 0\\ f_{44}(z)
\end{vmatrix} F_2(r), \Psi_3 = \begin{vmatrix}
0\\ f_{21}(z)\\ 0\\ f_{41}(z)\\ f_{12}(z)\\ 0\\ f_{32}(z)\\ 0\\ 0\\ f_{23}(z)\\ 0\\ f_{43}(z)\\ f_{14}(z)\\ 0\\ f_{34}(z)\\ 0
\end{vmatrix} F_3(r). \tag{12}
$$

It is convenient to introduce the following notations

$$
\begin{gathered}
\frac{1}{2}(f_{13}(z)+f_{31}(z)) = A(z), \quad \frac{1}{2}(f_{13}(z)-f_{31}(z)) = B(z),\\
\frac{1}{2}(f_{42}(z)-f_{24}(z)) = C(z), \quad \frac{1}{2}(f_{42}+f_{24}(z)) = D(z),\\
f_{13}(z) = A(z)+B(z), \qquad f_{31}(z) = A(z)-B(z),\\
f_{42}(z) = D(z)+C(z), \qquad f_{24}(z) = D(z)-C(z);
\end{gathered} \tag{13}
$$

then the three main constituents are presented as

$$\Psi_1 = \begin{vmatrix} f_{11}(z) \\ 0 \\ A(z) \\ 0 \\ 0 \\ 0 \\ 0 \\ C(z) \\ A(z) \\ 0 \\ f_{33}(z) \\ 0 \\ 0 \\ -C(z) \\ 0 \\ 0 \end{vmatrix} F_1, \quad \Psi_2 = \begin{vmatrix} 0 \\ 0 \\ -B(z) \\ 0 \\ 0 \\ f_{22}(z) \\ 0 \\ D(z) \\ B(z) \\ 0 \\ 0 \\ 0 \\ 0 \\ D(z) \\ 0 \\ f_{44}(z) \end{vmatrix} F_2(r), \quad \Psi_3 = \begin{vmatrix} 0 \\ f_{21}(z) \\ 0 \\ f_{41}(z) \\ f_{12}(z) \\ 0 \\ f_{32}(z) \\ 0 \\ 0 \\ f_{23}(z) \\ 0 \\ f_{43}(z) \\ f_{14}(z) \\ 0 \\ f_{34}(z) \\ 0 \end{vmatrix} F_3(r). \tag{14}$$

Further we derive the system of 1st order partial differential equations 4 with the shortening notations

$$\begin{aligned} &\frac{d}{dr} + \frac{m+1/2}{r} = a_{m+1/2}, \quad \frac{d}{dr} + \frac{m-1/2}{r} = a_{m-1/2}, \\ &\frac{d}{dr} - \frac{m+1/2}{r} = b_{m+1/2}, \quad \frac{d}{dr} - \frac{m-1/2}{r} = b_{m-1/2}, \end{aligned} \tag{15}$$

it reads

$$
\begin{aligned}
&a_{m-1/2} f_{41}(r,z) - iMf_{11}(r,z) + (-i(\epsilon - Ez) + \partial_z) f_{31}(r,z) = 0,\\
&a_{m+1/2} f_{42}(r,z) - iMf_{12}(r,z) + (-i(\epsilon - Ez) + \partial_z) f_{32}(r,z) = 0,\\
&a_{m-1/2} f_{43}(r,z) - iMf_{13}(r,z) + (-i(\epsilon - Ez) + \partial_z) f_{33}(r,z) = 0,\\
&a_{m+1/2} f_{44}(r,z) - iMf_{14}(r,z) + (-i(\epsilon - Ez) + \partial_z) f_{34}(r,z) = 0,\\
&b_{m-1/2} f_{31}(r,z) - iMf_{21}(r,z) + (-i(\epsilon - Ez) - \partial_z) f_{41}(r,z) = 0,\\
&b_{m+1/2} f_{32}(r,z) - iMf_{22}(r,z) + (-i(\epsilon - Ez) - \partial_z) f_{42}(r,z) = 0,\\
&b_{m-1/2} f_{33}(r,z) - iMf_{23}(r,z) + (-i(\epsilon - Ez) - \partial_z) f_{43}(r,z) = 0,\\
&b_{m+1/2} f_{34}(r,z) - iMf_{24}(r,z) + (-i(\epsilon - Ez) - \partial_z) f_{44}(r,z) = 0,\\
&-a_{m-1/2} f_{21}(r,z) - iMf_{31}(r,z) + (-i(\epsilon - Ez) - \partial_z) f_{11}(r,z) = 0,\\
&-a_{m+1/2} f_{22}(r,z) - iMf_{32}(r,z) + (-i(\epsilon - Ez) - \partial_z) f_{12}(r,z) = 0,\\
&-a_{m-1/2} f_{23}(r,z) - iMf_{33}(r,z) + (-i(\epsilon - Ez) - \partial_z) f_{13}(r,z) = 0,\\
&-a_{m+1/2} f_{24}(r,z) - iMf_{34}(r,z) + (-i(\epsilon - Ez) - \partial_z) f_{14}(r,z) = 0,\\
&-b_{m-1/2} f_{11}(r,z) - iMf_{41}(r,z) + (-i(\epsilon - Ez) + \partial_z) f_{21}(r,z) = 0,\\
&-b_{m+1/2} f_{12}(r,z) - iMf_{42}(r,z) + (-i(\epsilon - Ez) + \partial_z) f_{22}(r,z) = 0,\\
&-b_{m-1/2} f_{13}(r,z) - iMf_{43}(r,z) + (-i(\epsilon - Ez) + \partial_z) f_{23}(r,z) = 0,\\
&-b_{m+1/2} f_{14}(r,z) - iMf_{44}(r,z) + (-i(\epsilon - Ez) + \partial_z) f_{24}(r,z) = 0.
\end{aligned}
\tag{16}
$$

Let us transform these equations to other variables

$$
\begin{aligned}
&f_{13}(r,z) = A(r,z) + B(r,z), \quad f_{31}(r,z) = A(r,z) - B(r,z),\\
&f_{42}(r,z) = D(r,z) + C(r,z), \quad f_{24}(r,z) = D(r,z) - C(r,z), \textit{ and so on};
\end{aligned}
\tag{17}
$$

then we get

$$
\begin{aligned}
&1 \quad a_{m-1/2} f_{41}(r,z) - iMf_{11}(r,z) + (-i(\epsilon - Ez) + \partial_z)(A(r,z) - B(r,z)) = 0,\\
&2 \quad a_{m+1/2}(D(r,z) + C(r,z)) - iMf_{12}(r,z) + (-i(\epsilon - Ez) + \partial_z) f_{32}(r,z) = 0,\\
&3 \quad a_{m-1/2} f_{43}(r,z) - iM(A(r,z) + B(r,z)) + (-i(\epsilon - Ez) + \partial_z) f_{33}(r,z) = 0,\\
&4 \quad a_{m+1/2} f_{44}(r,z) - iMf_{14}(r,z) + (-i(\epsilon - Ez) + \partial_z) f_{34}(r,z) = 0,\\
&5 \quad b_{m-1/2}(A(r,z) - B(r,z)) - iMf_{21}(r,z) + (-i(\epsilon - Ez) - \partial_z) f_{41}(r,z) = 0,\\
&6 \quad b_{m+1/2} f_{32}(r,z) - iMf_{22}(r,z) + (-i(\epsilon - Ez) - \partial_z)(D(r,z) + C(r,z)) = 0,\\
&7 \quad b_{m-1/2} f_{33}(r,z) - iMf_{23}(r,z) + (-i(\epsilon - Ez) - \partial_z) f_{43}(r,z) = 0,\\
&8 \quad b_{m+1/2} f_{34}(r,z) - iM(D(r,z) - C(r,z)) + (-i(\epsilon - Ez) - \partial_z) f_{44}(r,z) = 0,\\
&9 \quad -a_{m-1/2} f_{21}(r,z) - iM(A(r,z) - B(r,z)) + (-i(\epsilon - Ez) - \partial_z) f_{11}(r,z) = 0,\\
&10 \quad -a_{m+1/2} f_{22}(r,z) - iMf_{32}(r,z) + (-i(\epsilon - Ez) - \partial_z) f_{12}(r,z) = 0,\\
&11 \quad -a_{m-1/2} f_{23} - iMf_{33} + (-i(\epsilon - Ez) - \partial_z)(A + B) = 0,\\
&12 - a_{m+1/2}(D(r,z) - C(r,z)) - iMf_{34}(r,z) + (-i(\epsilon - Ez) - \partial_z) f_{14}(r,z) = 0,\\
&13 \quad -b_{m-1/2} f_{11}(r,z) - iMf_{41}(r,z) + (-i(\epsilon - Ez) + \partial_z) f_{21}(r,z) = 0,\\
&14 \quad -b_{m+1/2} f_{12}(r,z) - iM(D(r,z) + C(r,z)) + (-i(\epsilon - Ez) + \partial_z) f_{22}(r,z) = 0,\\
&15 \quad -b_{m-1/2}(A(r,z) + B(r,z)) - iMf_{43}(r,z) + (-i(\epsilon - Ez) + \partial_z) f_{23}(r,z) = 0,\\
&16 \quad -b_{m+1/2} f_{14}(r,z) - iMf_{44}(r,z) + (-i(\epsilon - Ez) + \partial_z)(D(r,z) - C(r,z)) = 0.
\end{aligned}
\tag{18}
$$

It is convenient to apply the shortening notations $a_{m-1/2} = a_1$, $a_{m+1/2} = a_2$, $b_{m-1/2} = b_1$, $b_{m+1/2} = b_2$; so we obtain (combining equations as shown below)

$2) + 12)$

$$a_2(D(r,z) + C(r,z)) - iMf_{12}(r,z) + i(-(\epsilon - Ez) - i\partial_z) f_{32}(r,z) - a_2(D(r,z) - C(r,z)) -$$
$$-iMf_{34}(r,z) + i(-(\epsilon - Ez) + i\partial_z) f_{14}(r,z) = 0.$$

$2) - 12)$

$$a_2(D(r,z) + C(r,z)) - iMf_{12}(r,z) + i(-(\epsilon - Ez) - i\partial_z) f_{32}(r,z) +$$
$$+a_2(D(r,z) - C(r,z)) + iMf_{34}(r,z) - i(-(\epsilon - Ez) + i\partial_z) f_{14}(r,z) = 0.$$

$3) + 9)$

$$a_1 f_{43}(r,z) + i(-(\epsilon - Ez) - i\partial_z) f_{33}(r,z) - a_1 f_{21}(r,z) + i(-(\epsilon - Ez) + i\partial_z) f_{11}(r,z) -$$
$$-iM(A(r,z) + B(r,z)) - iM(A(r,z) - B) = 0,$$

$3) - 9)$

$$a_1 f_{43}(r,z) + i(-(\epsilon - Ez) - i\partial_z) f_{33}(r,z) + a_1 f_{21}(r,z) - i(-(\epsilon - Ez) + i\partial_z) f_{11}(r,z) -$$
$$-iM(A(r,z) + B(r,z)) + iM(A(r,z) - B(r,z)) = 0.$$

$5) + 15)$

$$b_{m-1/2}(A(r,z) - B(r,z)) - iMf_{21}(r,z) + i(-(\epsilon - Ez) + i\partial_z) f_{41}(r,z) -$$
$$-b_{m-1/2}(A(r,z) + B(r,z)) - iMf_{43}(r,z) + i(-(\epsilon - Ez) - i\partial_z) f_{23}(r,z) = 0,$$

$5) - 15)$

$$b_1(A(r,z) - B(r,z)) - iMf_{21}(r,z) + i(-(\epsilon - Ez) + i\partial_z) f_{41}(r,z) +$$
$$+b_1(A(r,z) + B(r,z)) + iMf_{43}(r,z) - i(-(\epsilon - Ez) - i\partial_z) f_{23}(r,z) = 0.$$

$8) + 14)$

$$b_2 f_{34}(r,z) + i(-(\epsilon - Ez) + i\partial_z) f_{44}(r,z) - b_2 f_{12} + i(-(\epsilon - Ez) - i\partial_z) f_{22}(r,z) -$$
$$-iM(D(r,z) - C(r,z)) - iM(D(r,z) + C(r,z)) = 0,$$

$8) - 14)$

$$b_2 f_{34} + i(-(\epsilon - Ez) + i\partial_z) f_{44} + b_2 f_{12} - i(-(\epsilon - Ez) - i\partial_z) f_{22} -$$
$$-iM(D(r,z) - C(r,z)) + iM(D(r,z) + C(r,z)) = 0.$$

Equations 1) and 22):

$$1) \qquad a_1 f_{41}(r,z) - i((\epsilon - Ez) + i\partial_z)(A(r,z) - B(r,z)) - iMf_{11}(r,z) = 0,$$
$$11) \qquad -a_1 f_{23}(r,z) - i((\epsilon - Ez) - i\partial_z)(A(r,z) + B(r,z)) - iMf_{33}(r,z) = 0;$$

let us use the notations

$$P = (\epsilon - Ez) + i\partial_z, \quad Q = (\epsilon - Ez) - i\partial_z, \quad P + Q = 2(\epsilon - Ez), \quad P - Q = 2i\partial_z,$$

then we have

$$a_1 f_{41}(r,z) - iP(A(r,z) - B(r,z)) - iMf_{11}(r,z) = 0,$$
$$-a_1 f_{23}(r,z) - iQ(A(r,z) + B(r,z)) - iMf_{33}(r,z) = 0;$$

we sum and subtract the last equations

$$a_1(f_{41}(r,z)-f_{23}(r,z))-i(P+Q)A(r,z)+i(P-Q)B(r,z)-iM(f_{11}(r,z)+f_{33}(r,z))=0,$$
$$a_1(f_{41}(r,z)+f_{23}(r,z))-i(P-Q)A(r,z)+i(P+Q)A(r,z)-iM(f_{11}(r,z)-f_{33}(r,z))=0,$$

or

$$a_1(f_{41}(r,z)-f_{23}(r,z))-2i(\epsilon-Ez)A(r,z)-2\partial_z B(r,z)-iM(f_{11}(r,z)+f_{33}(r,z))=0.$$
$$a_1(f_{41}(r,z)+f_{23}(r,z))+2\partial_z A(r,z)+2i(\epsilon-Ez)A(r,z)-iM(f_{11}(r,z)-f_{33}(r,z))=0.$$

Equations 6) and 16):

$$6)\qquad b_2 f_{32}(r,z)-i((\epsilon-Ez)-i\partial_z)(D(r,z)+C(r,z))-iMf_{22}(r,z)=0,$$
$$16)\qquad -b_2 f_{14}(r,z)-i((\epsilon-Ez)+i\partial_z)(D(r,z)-C(r,z))-iMf_{44}(r,z)=0,$$

or

$$b_2 f_{32}(r,z)-iQ(D(r,z)+C(r,z))-iMf_{22}(r,z)=0,$$
$$-b_2 f_{14}(r,z)-iP(D(r,z)-C(r,z))-iMf_{44}(r,z)=0;$$

we sum and subtract these equations

$$b_2(f_{32}(r,z)-f_{14}(r,z))-i(P+Q)D(r,z)+i(P-Q)C(r,z)-iM(f_{22}(r,z)+f_{44}(r,z))=0,$$
$$b_2(f_{32}(r,z)+f_{14}(r,z))+i(P-Q)D(r,z)-i(P+Q)C(r,z)-i(f_{22}(r,z)-f_{44}(r,z))=0,$$

or

$$b_2(f_{32}(r,z)-f_{14}(r,z))-2i(\epsilon-Ez)D(r,z)-2\partial_z C(r,z)-iM(f_{22}(r,z)+f_{44}(r,z))=0,$$
$$b_2(f_{32}(r,z)+f_{14}(r,z))-2\partial_z D(r,z)+2i(\epsilon-Ez)C(r,z)-i(f_{22}(r,z)-f_{44}(r,z))=0.$$

So we have four equations:

$$a_1(f_{41}(r,z)-f_{23}(r,z))-2i(\epsilon-Ez)A(r,z)-2\partial_z B(r,z)-iM(f_{11}(r,z)+f_{33}(r,z))=0.$$
$$a_1(f_{41}(r,z)+f_{23}(r,z))+2\partial_z A(r,z)+2i(\epsilon-Ez)A(r,z)-iM(f_{11}(r,z)-f_{33}(r,z))=0.$$
$$b_2(f_{32}(r,z)-f_{14}(r,z))-2i(\epsilon-Ez)D(r,z)-2\partial_z C(r,z)-iM(f_{22}(r,z)+f_{44}(r,z))=0,$$
$$b_2(f_{32}(r,z)+f_{14}(r,z))-2\partial_z D(r,z)+2i(\epsilon-Ez)C(r,z)-i(f_{22(r,z)}-f_{44}(r,z))=0.$$

The remaining (non-modified) equations are

$$4)\qquad a_2 f_{44}(r,z)+i(-(\epsilon-Ez)-i\partial_z)f_{34}(r,z)-iMf_{14}(r,z)=0,$$
$$7)\qquad b_1 f_{33}(r,z)+i(-(\epsilon-Ez)+i\partial_z)f_{43}(r,z)-iMf_{23}(r,z)=0,$$
$$10)\qquad -a_2 f_{22}(r,z)+i(-(\epsilon-Ez)+i\partial_z)f_{12}(r,z)-iMf_{32}(r,z)=0,$$
$$13)\qquad -b_1 f_{11}(r,z)+i(-(\epsilon-Ez)-i\partial_z)f_{21}(r,z)-iMf_{41}(r,z)=0.$$

Recall dependence of 16 variables on the basic functions $F_i(r)$:

$$\begin{gathered}\Psi_1,\ \ f_{11}, f_{33}, A, C \Rightarrow F_1(r); \qquad \Psi_2,\ \ f_{22}, f_{44}, B, D \Rightarrow F_2(r);\\ \Psi_3,\ \ f_{12}, f_{21}, f_{14}, f_{41}, f_{23}, f_{32}, f_{34}, f_{43} \Rightarrow\ \ F_3(3).\end{gathered} \tag{19}$$

Thus, we arrive at the system

$2)+12)$

$$a_2(D(r,z)+C(r,z))-iMf_{12}(r,z)-i((\epsilon-Ez)+i\partial_z)f_{32}(r,z)-a_2(D(r,z)-C(r,z))-$$
$$-iMf_{34}(r,z)-i((\epsilon-Ez)-i\partial_z)f_{14}(r,z)=0,$$

$2)-12)$

$$a_2(D(r,z)+C(r,z))-iMf_{12}(r,z)-i((\epsilon-Ez)+i\partial_z)f_{32}(r,z)+$$
$$+a_2(D(r,z)-C(r,z))+iMf_{34}(r,z)+i((\epsilon-Ez)-i\partial_z)f_{14}(r,z)=0,$$

$3)+9)$

$$a_1f_{43}(r,z)-i((\epsilon-Ez)+i\partial_z)f_{33}(r,z)-a_1f_{21}(r,z)-i((\epsilon-Ez)-i\partial_z)f_{11}(r,z)-$$
$$-iM(A(r,z)+B(r,z))-iM(A(r,z)-B(r,z))=0,$$

$3)-9)$

$$a_1f_{43}(r,z)-i((\epsilon-Ez)+i\partial_z)f_{33}(r,z)+a_1f_{21}(r,z)+i((\epsilon-Ez)-i\partial_z)f_{11}(r,z)-$$
$$-iM(A(r,z)+B(r,z))+iM(A(r,z)-B(r,z))=0.$$

$5)+15)$

$$b_{m-1/2}(A(r,z)-B(r,z))-iMf_{21}(r,z)-i((\epsilon-Ez)-i\partial_z)f_{41}(r,z)-$$
$$-b_{m-1/2}(A(r,z)+B(r,z))-iMf_{43}(r,z)-i((\epsilon-Ez)+i\partial_z)f_{23}(r,z)=0,$$

$5)-15)$

$$b_1(A(r,z)-B(r,z))-iMf_{21}(r,z)-i((\epsilon-Ez)-i\partial_z)f_{41}(r,z)+$$
$$+b_1(A(r,z)+B(r,z))+iMf_{43}(r,z)+i((\epsilon-Ez)+i\partial_z)f_{23}(r,z)=0,$$

$8)+14)$

$$b_2f_{34}(r,z)-i((\epsilon-Ez)-i\partial_z)f_{44}(r,z)-b_2f_{12}-i((\epsilon-Ez)+i\partial_z)f_{22}(r,z)-$$
$$-iM(D(r,z)-C(r,z))-iM(D(r,z)+C(r,z))=0,$$

$8)-14)$

$$b_2f_{34}(r,z)-i((\epsilon-Ez)-i\partial_z)f_{44}(r,z)+b_2f_{12}(r,z)+i((\epsilon-Ez)+i\partial_z)f_{22}(r,z)-$$
$$-iM(D(r,z)-C(r,z))+iM(D(r,z)+C(r,z))=0,$$

$$4)\qquad a_2f_{44}(r,z)-i((\epsilon-Ez)+i\partial_z)f_{34}(r,z)-iMf_{14}(r,z)=0,$$
$$7)\qquad b_1f_{33}(r,z)-i((\epsilon-Ez)-i\partial_z)f_{43}(r,z)-iMf_{23}(r,z)=0,$$
$$10\qquad -a_2f_{22}(r,z)-i((\epsilon-Ez)-i\partial_z)f_{12}(r,z)-iMf_{32}(r,z)=0,$$
$$13)\qquad -b_1f_{11}(r,z)-i((\epsilon-Ez)+i\partial_z)f_{21}(r,z)-iMf_{41}(r,z)=0.$$

$$a_1(f_{41}(r,z)-f_{23}(r,z))-2i(\epsilon-Ez)A(r,z)-2\partial_zB(r,z)-iM(f_{11}(r,z)+f_{33}(r,z))=0.$$
$$a_1(f_{41}(r,z)+f_{23}(r,z))+2\partial_zA(r,z)+2i(\epsilon-Ez)A(r,z)-iM(f_{11}(r,z)-f_{33}(r,z))=0,$$
$$b_2(f_{32}(r,z)-f_{14}(r,z))-2i(\epsilon-Ez)D(r,z)-2\partial_zC(r,z)-iM(f_{22}(r,z)+f_{44}(r,z))=0,$$
$$b_2(f_{32}(r,z)+f_{14}(r,z))-2\partial_zD(r,z)+2i(\epsilon-Ez)C(r,z)-i(f_{22}(r,z)-f_{44}(r,z))=0.$$

Remembering on definitions

$$a_{m-1/2}=a_1,\quad a_{m+1/2}=a_2,\quad b_{m-1/2}=b_1,\quad b_{m+1/2}=b_2;$$

we get

$2)+12)$

$$a_{m+1/2}D(r,z)+a_{m+1/2}C(r,z)-iMf_{12}(r,z)-i((\epsilon-Ez)+i\partial_z)f_{32}(r,z)-a_{m+1/2}D(r,z)+a_{m+1/2}C(r,z)-$$

$$-iMf_{34}(r,z)-i((\epsilon-Ez)-i\partial_z)f_{14}(r,z)=0,$$

2) − 12)

$$a_{m+1/2}D(r,z)+a_{m+1/2}C(r,z)-iMf_{12}(r,z)-i((\epsilon-Ez)+i\partial_z)f_{32}(r,z)+$$
$$+a_{m+1/2}D(r,z)-a_{m+1/2}C(r,z)+iMf_{34}(r,z)+i((\epsilon-Ez)-i\partial_z)f_{14}(r,z)=0,$$

3) + 9)

$$a_{m-1/2}f_{43}(r,z)-i((\epsilon-Ez)+i\partial_z)f_{33}(r,z)-a_{m-1/2}f_{21}(r,z)-$$
$$-i((\epsilon-Ez)-i\partial_z)f_{11}(r,z)-2iMA(r,z)=0,$$

3) − 9)

$$a_{m-1/2}f_{43}(r,z)-i((\epsilon-Ez)+i\partial_z)f_{33}(r,z)+a_{m-1/2}f_{21}(r,z)+$$
$$+i((\epsilon-Ez)-i\partial_z)f_{11}(r,z)-2iMB(r,z)=0.$$

5) + 15)

$$b_{m-1/2}A(r,z)-b_{m-1/2}B(r,z)-iMf_{21}(r,z)-i((\epsilon-Ez)-i\partial_z)f_{41}(r,z)-$$
$$-b_{m-1/2}A(r,z)-b_{m-1/2}B(r,z)-iMf_{43}(r,z)-i((\epsilon-Ez)+i\partial_z)f_{23}(r,z)=0,$$

5) − 15)

$$b_{m-1/2}A(r,z)-b_{m-1/2}B(r,z)-iMf_{21}(r,z)-i((\epsilon-Ez)-i\partial_z)f_{41}(r,z)+$$
$$+b_{m-1/2}A(r,z)+b_{m-1/2}B(r,z)+iMf_{43}(r,z)+i((\epsilon-Ez)+i\partial_z)f_{23}(r,z)=0,$$

8) + 14)

$$b_{m+1/2}f_{34}(r,z)-i((\epsilon-Ez)-i\partial_z)f_{44}(r,z)-b_{m+1/2}f_{12}-i((\epsilon-Ez)+i\partial_z)f_{22}(r,z)-2iMD(r,z)=0,$$

8) − 14)

$$b_{m+1/2}f_{34}-i((\epsilon-Ez)-i\partial_z)f_{44}+b_{m+1/2}f_{12}+i((\epsilon-Ez)+i\partial_z)f_{22}+2iMC(r,z)=0,$$

4) $a_{m+1/2}f_{44}(r,z)-i((\epsilon-Ez)+i\partial_z)f_{34}(r,z)-iMf_{14}(r,z)=0,$

7) $b_{m-1/2}f_{33}(r,z)-i((\epsilon-Ez)-i\partial_z)f_{43}(r,z)-iMf_{23}(r,z)=0,$

10 $-a_{m+1/2}f_{22}(r,z)-i((\epsilon-Ez)-i\partial_z)f_{12}(r,z)-iMf_{32}(r,z)=0,$

13) $-b_{m-1/2}f_{11}(r,z)-i((\epsilon-Ez)+i\partial_z)f_{21}(r,z)-iMf_{41}(r,z)=0,$

$$a_{m-1/2}(f_{41}(r,z)-f_{23}(r,z))-2i(\epsilon-Ez)A(r,z)-2\partial_z B(r,z)-iM(f_{11}(r,z)+f_{33}(r,z))=0,$$
$$a_{m-1/2}(f_{41}(r,z)+f_{23}(r,z))+2\partial_z A(r,z)+2i(\epsilon-Ez)A(r,z)-iM(f_{11}(r,z)-f_{33}(r,z))=0,$$
$$b_{m+1/2}(f_{32}(r,z)-f_{14}(r,z))-2i(\epsilon-Ez)D(r,z)-2\partial_z C(r,z)-iM(f_{22}(r,z)+f_{44}(r,z))=0,$$
$$b_{m+1/2}(f_{32}(r,z)+f_{14}(r,z))-2\partial_z D(r,z)+2i(\epsilon-Ez)C(r,z)-i(f_{22}(r,z)-f_{44}(r,z))=0,$$

Taking in mind dependence of 16 variables on the three basic functions: $F_i(r)$:

$$\begin{aligned}
&f_{11}, f_{33}, A, C \;\Rightarrow\; f_{11}(z)F_1(r),\quad f_{33}(z)F_1(r),\quad A(z)F_1(r),\quad C(z)F_1(r);\\
&f_{22}, f_{44}, B, D \;\Rightarrow\; f_{22}(z)F_2(r),\quad f_{44}(z)F_2(r),\quad B(z)F_2(r),\quad D(z)F_2(r);\\
&f_{12}, f_{21}, f_{14}, f_{41} \;\Rightarrow\; f_{12}(z)F_3(r),\quad f_{21}(z)F_3(r),\quad f_{14}(z)F_3(r),\quad f_{41}(z)F_3(r)\\
&f_{23}, f_{32}, f_{34}, f_{43} \;\Rightarrow\; f_{23}(z)F_3(r),\quad f_{32}(z)F_3(r),\quad f_{34}(z)F_3(r),\quad f_{43}(z)F_3(r),
\end{aligned} \tag{20}$$

we transform the above equations to the form

$2)+12)$

$$a_{m+1/2}D(z)F_2(r)+a_{m+1/2}C(z)F_1(r)-iMf_{12}(z)F_3(r)-i((\epsilon-Ez)+i\partial_z)f_{32}(z)F_3(r)-$$
$$-a_{m+1/2}D(z)F_2(r)+a_{m+1/2}C(z)F_1(r)-iMf_{34}(z)F_3(r)-i((\epsilon-Ez)-i\partial_z)f_{14}(z)F_3(r)=0,$$

$2)-12)$

$$a_{m+1/2}D(z)F_2(r)+a_{m+1/2}C(z)F_1(r)-iMf_{12}(z)F_3(r)-i((\epsilon-Ez)+i\partial_z)f_{32}(z)F_3(r)+$$
$$+a_{m+1/2}D(z)F_2(r)-a_{m+1/2}C(z)F_1(r)+iMf_{34}(z)F_3(r)+i((\epsilon-Ez)-i\partial_z)f_{14}(z)F_3(r)=0,$$

$3)+9)$

$$a_{m-1/2}f_{43}(z)F_3(r)-i((\epsilon-Ez)+i\partial_z)f_{33}(z)F_1(r)-a_{m-1/2}f_{21}(z)F_3(r)-$$
$$-i((\epsilon-Ez)-i\partial_z)f_{11}(z)F_1(r)-2iMA(z)F_1(r)=0,$$

$3)-9)$

$$a_{m-1/2}f_{43}(z)F_3(r)-i((\epsilon-Ez)+i\partial_z)f_{33}(z)F_1(r)+a_{m-1/2}f_{21}(z)F_3(r)+$$
$$+i((\epsilon-Ez)-i\partial_z)f_{11}(z)F_1(r)-2iMB(z)F_2(r)=0,$$

$5)+15)$

$$b_{m-1/2}A(z)F_1(r)-b_{m-1/2}B(z)F_2(r)-iMf_{21}(z)F_3(r)-i((\epsilon-Ez)-i\partial_z)f_{41}(z)F_3(r)-$$
$$-b_{m-1/2}A(z)F_1(r)-b_{m-1/2}B(z)F_2(r)-iMf_{43}(z)F_3(r)-i((\epsilon-Ez)+i\partial_z)f_{23}(z)F_3(r)=0,$$

$5)-15)$

$$b_{m-1/2}A(z)F_1(r)-b_{m-1/2}B(z)F_2(r)-iMf_{21}(z)F_3(r)-i((\epsilon-Ez)-i\partial_z)f_{41}(z)F_3(r)+$$
$$+b_{m-1/2}A(z)F_1(r)+b_{m-1/2}B(z)F_2(r)+iMf_{43}(z)F_3(r)+i((\epsilon-Ez)+i\partial_z)f_{23}(z)F_3(r)=0,$$

$8)+14)$

$$b_{m+1/2}f_{34}(z)F_3(r)-i((\epsilon-Ez)-i\partial_z)f_{44}(z)F_2(r)-b_{m+1/2}f_{12}(z)F_3(r)-$$
$$-i((\epsilon-Ez)+i\partial_z)f_{22}(z)F_2(r)-2iMD(z)F_2(r)=0,$$

$8)-14)$

$$b_{m+1/2}f_{34}(z)F_3(r)-i((\epsilon-Ez)-i\partial_z)f_{44}(z)F_2(r)+b_{m+1/2}f_{12}(z)F_3(r)+$$
$$+i((\epsilon-Ez)+i\partial_z)f_{22}(z)F_2(r)+2iMC(z)F_1(r)=0,$$

$$4)\qquad a_{m+1/2}f_{44}(z)F_2(r)-i((\epsilon-Ez)+i\partial_z)f_{34}(z)F_3(r)-iMf_{14}(z)F_3(r)=0,$$
$$7)\qquad b_{m-1/2}f_{33}(z)F_1(r)-i((\epsilon-Ez)-i\partial_z)f_{43}(z)F_3(r)-iMf_{23}(z)F_3(r)=0,$$
$$10\qquad -a_{m+1/2}f_{22}(z)F_2(r)-i((\epsilon-Ez)-i\partial_z)f_{12}(z)F_3(r)-iMf_{32}(z)F_3(r)=0,$$
$$13)\qquad -b_{m-1/2}f_{11}(z)F_1(r)-i((\epsilon-Ez)+i\partial_z)f_{21}(z)F_3(r)-iMf_{41}(z)F_3(r)=0,$$
$$a_{m-1/2}(f_{41}(z)-f_{23}(z))F_3(r)-2i(\epsilon-Ez)A(z)F_1(r)-2\partial_zB(z)F_2(r)-iM(f_{11}(z)+f_{33}(z))F_1(r)=0.$$

$$a_{m-1/2}(f_{41}(z)+f_{23}(z))F_3(r)+2\partial_zA(z)F_1(r)+2i(\epsilon-Ez)A(z)F_1(r)-iM(f_{11}(z)-f_{33}(z))F_1(r)=0,$$
$$b_{m+1/2}(f_{32}(z)-f_{14}(z))F_3(r)-2i(\epsilon-Ez)D(z)F_2(r)-2\partial_zC(z)F_1(r)-iM(f_{22}(z)+f_{44}(z))F_2(r)=0,$$

$$b_{m+1/2}(f_{32}(z)+f_{14}(z))F_3(r)-2\partial_zD(z)F_2(r)+2i(\epsilon-Ez)C(z)F_1(r)-i(f_{22}(z)-f_{44}(z))F_2(r)=0.$$

It is convenient to collect equations into the three group (following the type of the basic function, below we should set $B(z)=0, C(z)=0$):

I

$$f_{43}(z)a_{m-1/2}F_3(r) - i((\epsilon - Ez) + i\partial_z) f_{33}(z)F_1(r) - f_{21}(z)a_{m-1/2}F_3(r) -$$
$$-i((\epsilon - Ez) - i\partial_z) f_{11}(z)F_1(r) - 2iMA(z)F_1(r) = 0,$$
$$f_{43}(z)a_{m-1/2}F_3(r) - i((\epsilon - Ez) + i\partial_z) f_{33}(z)F_1(r) + f_{21}(z)a_{m-1/2}F_3(r) +$$
$$+i((\epsilon - Ez) - i\partial_z) f_{11}(z)F_1(r) - 2iMB(z)F_2(r) = 0.$$
$$(f_{41}(z) - f_{23}(z))a_{m-1/2}F_3(r) - 2i(\epsilon - Ez)A(z)F_1(r) - 2\partial_z B(z)F_2(r) -$$
$$-iM(f_{11}(z) + f_{33}(z))F_1(r) = 0.$$
$$(f_{41}(z) + f_{23}(z))a_{m-1/2}F_3(r) + 2\partial_z A(z)F_1(r) + 2i(\epsilon - Ez)A(z)F_1(r) -$$
$$-iM(f_{11}(z) - f_{33}(z))F_1(r) = 0,$$

II

$$f_{34}(z)b_{m+1/2}F_3(r) - i((\epsilon - Ez) - i\partial_z) f_{44}(z)F_2(r) - f_{12}(z)b_{m+1/2}F_3(r) -$$
$$-i((\epsilon - Ez) + i\partial_z) f_{22}(z)F_2(r) - 2iMD(z)F_2(r) = 0,$$
$$f_{34}(z)b_{m+1/2}F_3(r) - i((\epsilon - Ez) - i\partial_z) f_{44}(z)F_2(r) + f_{12}(z)b_{m+1/2}F_3(r) +$$
$$+i((\epsilon - Ez) + i\partial_z) f_{22}(z)F_2(r) + 2iMC(z)F_1(r) = 0,$$
$$(f_{32}(z) - f_{14}(z))b_{m+1/2}F_3(r) - 2i(\epsilon - Ez)D(z)F_2(r) - 2\partial_z C(z)F_1(r) -$$
$$-iM(f_{22}(z) + f_{44}(z))F_2(r) = 0,$$
$$(f_{32}(z) + f_{14}(z))b_{m+1/2}F_3(r) - 2\partial_z D(z)F_2(r) + 2i(\epsilon - Ez)C(z)F_1(r) -$$
$$-i(f_{22}(z) - f_{44}(z))F_2(r) = 0,$$

III

$$D(z)a_{m+1/2}F_2(r) + C(z)a_{m+1/2}F_1(r) - iMf_{12}(z)F_3(r) - i((\epsilon - Ez) + i\partial_z) f_{32}(z)F_3(r) -$$
$$-D(z)a_{m+1/2}F_2(r) + C(z)a_{m+1/2}F_1(r) - iMf_{34}(z)F_3(r) - i((\epsilon - Ez) - i\partial_z) f_{14}(z)F_3(r) = 0,$$
$$D(z)a_{m+1/2}F_2(r) + C(z)a_{m+1/2}F_1(r) - iMf_{12}(z)F_3(r) - i((\epsilon - Ez) + i\partial_z) f_{32}(z)F_3(r) +$$
$$+D(z)a_{m+1/2}F_2(r) - C(z)a_{m+1/2}F_1(r) + iMf_{34}(z)F_3(r) + i((\epsilon - Ez) - i\partial_z) f_{14}(z)F_3(r) = 0,$$
$$A(z)b_{m-1/2}F_1(r) - B(z)b_{m-1/2}F_2(r) - iMf_{21}(z)F_3(r) - i((\epsilon - Ez) - i\partial_z) f_{41}(z)F_3(r) -$$
$$-A(z)b_{m-1/2}F_1(r) - B(z)b_{m-1/2}F_2(r) - iMf_{43}(z)F_3(r) - i((\epsilon - Ez) + i\partial_z) f_{23}(z)F_3(r) = 0,$$
$$A(z)b_{m-1/2}F_1(r) - B(z)b_{m-1/2}F_2(r) - iMf_{21}(z)F_3(r) - i((\epsilon - Ez) - i\partial_z) f_{41}(z)F_3(r) +$$
$$+A(z)b_{m-1/2}F_1(r) + B(z)b_{m-1/2}F_2(r) + iMf_{43}(z)F_3(r) + i((\epsilon - Ez) + i\partial_z) f_{23}(z)F_3(r) = 0,$$
$$f_{44}(z)a_{m+1/2}F_2(r) - i((\epsilon - Ez) + i\partial_z) f_{34}(z)F_3(r) - iMf_{14}(z)F_3(r) = 0,$$
$$f_{33}(z)b_{m-1/2}F_1(r) - i((\epsilon - Ez) - i\partial_z) f_{43}(z)F_3(r) - iMf_{23}(z)F_3(r) = 0,$$
$$-f_{22}(z)a_{m+1/2}F_2(r) - i((\epsilon - Ez) - i\partial_z) f_{12}(z)F_3(r) - iMf_{32}(z)F_3(r) = 0,$$
$$-f_{11}(z)b_{m-1/2}F_1(r) - i((\epsilon - Ez) + i\partial_z) f_{21}(z)F_3(r) - iMf_{41}(z)F_3(r) = 0.$$

In accordance with the general method by Fedorov – Gronskiy, we should impose the relevant differential constraints which permit us to transform all equations to algebraic form

$$\begin{aligned} &B(z) = 0, \quad C(z) = 0, \\ &a_{m-1/2}F_3(r) = C_1F_1(r), \quad b_{m+1/2}F_3(r) = C_2F_2(r), \\ &b_{m-1/2}F_1(r) = C_3F_3(r), \quad a_{m+1/2}F_2(r) = C_4F_3(r); \end{aligned} \tag{21}$$

in this way we obtain

I

$$f_{43}(z)C_1F_1(r) - i((\epsilon - Ez) + i\partial_z) f_{33}(z)F_1(r) - f_{21}(z)C_1F_1(r) -$$
$$-i((\epsilon - Ez) - i\partial_z) f_{11}(z)F_1(r) - 2iMA(z)F_1(r) = 0,$$
$$f_{43}(z)C_1F_1(r) - i((\epsilon - Ez) + i\partial_z) f_{33}(z)F_1(r) + f_{21}(z)C_1F_1(r) +$$
$$+i((\epsilon - Ez) - i\partial_z) f_{11}(z)F_1(r) = 0,$$
$$(f_{41}(z) - f_{23}(z))C_1F_1(r) - 2i(\epsilon - Ez)A(z)F_1(r) - iM(f_{11}(z) + f_{33}(z))F_1(r) = 0,$$
$$(f_{41}(z) + f_{23}(z))C_1F_1(r) + 2\partial_z A(z)F_1(r) + 2i(\epsilon - Ez)A(z)F_1(r) - iM(f_{11}(z) - f_{33}(z))F_1(r) = 0,$$

II

$$f_{34}(z)C_2F_2(r) - i((\epsilon - Ez) - i\partial_z) f_{44}(z)F_2(r) - f_{12}(z)C_2F_2(r) -$$
$$-i((\epsilon - Ez) + i\partial_z) f_{22}(z)F_2(r) - 2iMD(z)F_2(r) = 0,$$
$$f_{34}(z)C_2F_2(r) - i((\epsilon - Ez) - i\partial_z) f_{44}(z)F_2(r) + f_{12}(z)C_2F_2(r) +$$
$$+i((\epsilon - Ez) + i\partial_z) f_{22}(z)F_2(r) = 0,$$
$$(f_{32}(z) - f_{14}(z))C_2F_2(r) - 2i(\epsilon - Ez)D(z)F_2(r) - iM(f_{22}(z) + f_{44}(z))F_2(r) = 0,$$
$$(f_{32}(z) + f_{14}(z))C_2F_2(r) - 2\partial_z D(z)F_2(r) - i(f_{22}(z) - f_{44}(z))F_2(r) = 0,$$

III

$$D(z)C_4F_3(r) - iMf_{12}(z)F_3(r) - i((\epsilon - Ez) + i\partial_z) f_{32}(z)F_3(r) -$$
$$-D(z)C_4F_3(r) - iMf_{34}(z)F_3(r) - i((\epsilon - Ez) - i\partial_z) f_{14}(z)F_3(r) = 0,$$
$$D(z)C_4F_3(r) - iMf_{12}(z)F_3(r) - i((\epsilon - Ez) + i\partial_z) f_{32}(z)F_3(r) +$$
$$+D(z)C_4F_3(r) + iMf_{34}(z)F_3(r) + i((\epsilon - Ez) - i\partial_z) f_{14}(z)F_3(r) = 0,$$
$$A(z)C_3F_3(r) - iMf_{21}(z)F_3(r) - i((\epsilon - Ez) - i\partial_z) f_{41}(z)F_3(r) -$$
$$-A(z)C_3F_3(r) - iMf_{43}(z)F_3(r) - i((\epsilon - Ez) + i\partial_z) f_{23}(z)F_3(r) = 0,$$
$$A(z)C_3F_3(r) - iMf_{21}(z)F_3(r) - i((\epsilon - Ez) - i\partial_z) f_{41}(z)F_3(r) +$$
$$+A(z)C_3F_3(r) + iMf_{43}(z)F_3(r) + i((\epsilon - Ez) + i\partial_z) f_{23}(z)F_3(r) = 0,$$
$$f_{44}(z)C_4F_3(r) - i((\epsilon - Ez) + i\partial_z) f_{34}(z)F_3(r) - iMf_{14}(z)F_3(r) = 0,$$
$$f_{33}(z)C_3F_3(r) - i((\epsilon - Ez) - i\partial_z) f_{43}(z)F_3(r) - iMf_{23}(z)F_3(r) = 0,$$
$$-f_{22}(z)C_4F_3(r) - i((\epsilon - Ez) - i\partial_z) f_{12}(z)F_3(r) - iMf_{32}(z)F_3(r) = 0,$$
$$-f_{11}(z)C_3F_3(r) - i((\epsilon - Ez) + i\partial_z) f_{21}(z)F_3(r) - iMf_{41}(z)F_3(r) = 0.$$

Reducing the total multipliers, we derive the system of differentuial equations in the variable z:

I

$$C_1 f_{43} - i((\epsilon - Ez) + i\partial_z) f_{33} - C_1 f_{21} - i((\epsilon - Ez) - i\partial_z) f_{11} - 2iMA = 0,$$
$$C_1 f_{43} - i((\epsilon - Ez) + i\partial_z) f_{33} + C_1 f_{21} + i((\epsilon - Ez) - i\partial_z) f_{11} = 0,$$
$$C_1(f_{41} - f_{23}) - 2i(\epsilon - Ez)A - iM(f_{11} + f_{33}) = 0,$$
$$C_1(f_{41} + f_{23}) + 2\partial_z A + 2i(\epsilon - Ez)A - iM(f_{11} - f_{33}) = 0,$$

II

$$C_2 f_{34} - i((\epsilon - Ez) - i\partial_z) f_{44} - C_2 f_{12} - i((\epsilon - Ez) + i\partial_z) f_{22} - 2iMD = 0,$$
$$C_2 f_{34} - i((\epsilon - Ez) - i\partial_z) f_{44} + C_2 f_{12} + i((\epsilon - Ez) + i\partial_z) f_{22} = 0,$$
$$C_2(f_{32} - f_{14}) - 2i(\epsilon - Ez)D - iM(f_{22} + f_{44}) = 0,$$
$$C_2(f_{32} + f_{14}) - 2\partial_z D - iM(f_{22} - f_{44}) = 0,$$

III

$$C_4 D - iMf_{12} - i((\epsilon - Ez) + i\partial_z) f_{32} - C_4 D - iMf_{34} - i((\epsilon - Ez) - i\partial_z) f_{14} = 0,$$
$$C_4 D - iMf_{12} - i((\epsilon - Ez) + i\partial_z) f_{32} + iMf_{34} + i((\epsilon - Ez) - i\partial_z) f_{14} = 0,$$
$$C_3 A - iMf_{21} - i((\epsilon - Ez) - i\partial_z) f_{41} - C_3 A - iMf_{43} - i((\epsilon - Ez) + i\partial_z) f_{23} = 0,$$
$$C_3 A - iMf_{21} - i((\epsilon - Ez) - i\partial_z) f_{41} + C_3 A + iMf_{43} + i((\epsilon - Ez) + i\partial_z) f_{23} = 0,$$
$$C_4 f_{44} - i((\epsilon - Ez) + i\partial_z) f_{34} - iMf_{14} = 0,$$
$$C_3 f_{33} - i((\epsilon - Ez) - i\partial_z) f_{43} - iMf_{23} = 0,$$
$$-C_4 f_{22} - i((\epsilon - Ez) - i\partial_z) f_{12} - iMf_{32} = 0,$$
$$-C_3 f_{11} - i((\epsilon - Ez) + i\partial_z) f_{21} - iMf_{41} = 0.$$

2. Differential constraints

Let us study the above differential relations

$$\begin{gathered} a_{m-1/2} F_3 = C_1 F_1, \quad b_{m-1/2} F_1 = C_3 F_3, \; let \; C_3 = C_1, \\ b_{m+1/2} F_3 = C_2 F_2, \quad a_{m+1/2} F_2 = C_4 F_3, \; let \; C_4 = C_2. \end{gathered} \tag{22}$$

We readily derive the 2nd order equations

$$\begin{gathered} (a_{m-1/2} b_{m-1/2} - C_1^2) F_1 = 0, \quad (b_{m-1/2} a_{m-1/2} - C_1^2) F_3 = 0, \\ (b_{m+1/2} a_{m+1/2} - C_2^2) F_2 = 0, \quad (a_{m+1/2} b_{m+1/2} - C_2^2) F_3 = 0. \end{gathered} \tag{23}$$

Taking in mind the definitions

$$\frac{d}{dr} + \frac{m+1/2}{r} = a_{m+1/2}, \quad \frac{d}{dr} + \frac{m-1/2}{r} = a_{m-1/2},$$
$$\frac{d}{dr} - \frac{m+1/2}{r} = b_{m+1/2}, \quad \frac{d}{dr} - \frac{m-1/2}{r} = b_{m-1/2},$$

we arrive at four differential equations for 3 functions:

$$\left(\frac{d^2}{dr^2} - \frac{(4(m-2)m+3)}{4r^2} - C_1^2\right) F_1 = 0, \quad \left(\frac{d^2}{dr^2} + \frac{\left(1-4m^2\right)}{4r^2} - C_1^2\right) F_3 = 0,$$
$$\left(\frac{d^2}{dr^2} - \frac{(4m(m+2)+3)}{4r^2} - C_2^2\right) F_2 = 0, \quad \left(\frac{d^2}{dr^2} + \frac{\left(1-4m^2\right)}{4r^2} - C_2^2\right) F_3 = 0.$$

due to 2nd and 4th equations we have evident constraint $C_1^2 = C_2^2 = C^2$, so obtain

$$\begin{gathered} \left(\frac{d^2}{dr^2} - \frac{(m-1)^2 - 1/4}{r^2} - C^2\right) F_1 = 0, \\ \left(\frac{d^2}{dr^2} - \frac{(m+1)^2 - 1/4}{r^2} - C^2\right) F_2 = 0, \\ \left(\frac{d^2}{dr^2} - \frac{m^2 - 1/4}{r^2} - C^2\right) F_3 = 0. \end{gathered} \tag{24}$$

From the last equation, from the needed asymptotic at infinity $r \to +\infty$ we conclude that parameter C should be imaginary $C = iX$, so we obtain

$$e^{\pm Cr} \sim e^{+iXr}, e^{-iXr}.$$

Let us consider in detail the third equation. In the variable $x = Cr = iXr$, it reads

$$(\frac{d^2}{dx^2} - \frac{m^2 - 1/4}{x^2} - 1)F_3 = 0.$$

Its solutions are searched in the form $F_3 = x^A e^{Bx} f_3(x)$; after simple calculation we derive an equation for function $f_3(x)$:

$$\frac{d^2}{dx^2} f_3 + [\frac{2A}{x} + 2B]\frac{d}{dx} f_3 + [\frac{A(A-1) - (m^2 - 1/4)}{x^2} + (B^2 - 1) + \frac{2AB}{x}] f_3 = 0.$$

Imposing the the evident constraints, we get

$$A(A-1) - (m^2 - 1/4) = 0 \quad \Rightarrow \quad A = \pm |m| + \frac{1}{2},$$

$$B = \pm 1, \quad F_3 = x^A e^{Bx} f_3(x) = (iXr)^{\pm|m|+1/2} e^{\pm iXr} f_3(x);$$

so we arrive at the simpler equation for $f_3(x)$:

$$x\frac{d^2}{dx^2} f_3 + (2A + 2Bx)\frac{d}{dx} f_3 + 2ABf_3 = 0.$$

In the variable $2Bx = -y$, it reads

$$2Bx = -y, \qquad y\frac{d^2}{dy^2} f_3 + (2A - y)\frac{d}{dy} f_3 - Af_3 = 0,$$

which is identified with the confluent hypergeometric equation

$$(y\frac{d^2}{dy^2} + (c - y)\frac{d}{dy} - a)F(a, c; y) = 0;$$

its solutions may be related with Bessel functions. For definiteness, we set

$$B = -1, \quad y = 2iXr, \quad F(a, c; 2iXr), \quad a = A = \pm |m| + \frac{1}{2}, \quad c = 2A = 2(\pm |m| + \frac{1}{2}); \qquad (25)$$

depending on the sign of the quantum number m, we have two different possibilities.

Two other cases differ only in formal changes

$$m^2 - \frac{1}{4} \quad \Rightarrow \quad (m-1)^2 - \frac{1}{4}, \quad (m+1)^2 - \frac{1}{4}. \qquad (26)$$

3. System of equations in z-variable

From studying the differential constraints, follow identities

$$C_1 = C_2 = C_3 = C_4 = C;$$

then we have the system

I

$$cf_{43} - i((\epsilon - Ez) + i\partial_z) f_{33} - cf_{21} - i((\epsilon - Ez) - i\partial_z) f_{11} - 2iMA = 0,$$

$$cf_{43} - i((\epsilon - Ez) + i\partial_z) f_{33} + cf_{21} + i((\epsilon - Ez) - i\partial_z) f_{11} = 0,$$

$$c(f_{41} - f_{23}) - 2i(\epsilon - Ez)A - iM(f_{11} + f_{33}) = 0,$$

$$c(f_{41} + f_{23}) + 2\partial_z A + 2i(\epsilon - Ez)A - iM(f_{11} - f_{33}) = 0,$$

II

$$cf_{34}-i((\epsilon-Ez)-i\partial_z)f_{44}-cf_{12}-i((\epsilon-Ez)+i\partial_z)f_{22}-2iMD=0,$$
$$cf_{34}-i((\epsilon-Ez)-i\partial_z)f_{44}+cf_{12}+i((\epsilon-Ez)+i\partial_z)f_{22}=0,$$
$$c(f_{32}-f_{14})-2i(\epsilon-Ez)D-iM(f_{22}+f_{44})=0,$$
$$c(f_{32}+f_{14})-2\partial_z D-iM(f_{22}-f_{44})=0,$$

III

$$cD-iMf_{12}-i((\epsilon-Ez)+i\partial_z)f_{32}-cD-iMf_{34}-i((\epsilon-Ez)-i\partial_z)f_{14}=0,$$
$$cD-iMf_{12}-i((\epsilon-Ez)+i\partial_z)f_{32}+iMf_{34}+i((\epsilon-Ez)-i\partial_z)f_{14}=0,$$
$$cA-iMf_{21}-i((\epsilon-Ez)-i\partial_z)f_{41}-cA-iMf_{43}-i((\epsilon-Ez)+i\partial_z)f_{23}=0,$$
$$cA-iMf_{21}-i((\epsilon-Ez)-i\partial_z)f_{41}+cA+iMf_{43}+i((\epsilon-Ez)+i\partial_z)f_{23}=0,$$
$$cf_{44}-i((\epsilon-Ez)+i\partial_z)f_{34}-iMf_{14}=0,$$
$$cf_{33}-i((\epsilon-Ez)-i\partial_z)f_{43}-iMf_{23}=0,$$
$$-cf_{22}-i((\epsilon-Ez)-i\partial_z)f_{12}-iMf_{32}=0,$$
$$-cf_{11}-i((\epsilon-Ez)+i\partial_z)f_{21}-iMf_{41}=0.$$

Solving the first 8 equations with respect to the variables f_{12}, f_{21}, f_{23}, f_{32}, f_{14}, f_{41}, f_{34}, f_{43}, we obtain

$$f_{12}=\frac{-iMD+if_{22}(Ez-\epsilon)+f_{22}'}{c},\quad f_{21}=\frac{i\left(-MA+f_{11}(Ez-\epsilon)+if_{11}'\right)}{c},$$
$$f_{23}=\frac{i\left(iA'+2A(Ez-\epsilon)-Mf_{33}\right)}{c},\quad f_{32}=\frac{D'-iD(Ez-\epsilon)+iMf_{22}}{c},$$
$$f_{14}=\frac{D'+iD(Ez-\epsilon)-iMf_{44}}{c},\quad f_{41}=\frac{-A'+iMf_{11}}{c}, \tag{27}$$
$$f_{34}=\frac{f_{44}'+i\left(MD+f_{44}(\epsilon-Ez)\right)}{c},\quad f_{43}=\frac{i\left(MA+f_{33}(\epsilon-Ez)+if_{33}'\right)}{c}.$$

Substituting them into remaining 8 equations, we derive

$$-\frac{2iED}{c}=0,$$
$$\frac{D\left(c^2+2(\epsilon-Ez)^2-2M^2\right)+2D''}{c}=0,$$
$$-\frac{2A\left(E\left(Ez^2-i\right)-2Ez\epsilon+\epsilon^2\right)}{c}=0,$$
$$\frac{2\left(A''-iA'(Ez-\epsilon)+A\left(c^2+E\left(Ez^2-i\right)-2Ez\epsilon-M^2+\epsilon^2\right)\right)}{c}=0,$$
$$\frac{f_{44}''+f_{44}\left(c^2+E\left(Ez^2-i\right)-2Ez\epsilon-M^2+\epsilon^2\right)}{c}=0,$$
$$\frac{MA(Ez-\epsilon)+f_{33}\left(c^2+E\left(Ez^2+i\right)-2Ez\epsilon-M^2+\epsilon^2\right)+f_{33}''}{c}=0,$$

$$-\frac{f_{22}{}'' + f_{22}\left(c^2 + E\left(Ez^2 + i\right) - 2Ez\epsilon - M^2 + \epsilon^2\right)}{c} = 0,$$

$$-\frac{MA(\epsilon - Ez) + f_{11}\left(c^2 + E\left(Ez^2 - i\right) - 2Ez\epsilon - M^2 + \epsilon^2\right) + f_{11}{}''}{c} = 0,$$

whence it follows

$$D = 0,$$

$$2A\Big(E\left(Ez^2 - i\right) - 2Ez\epsilon + \epsilon^2\Big) = 0 \quad \Rightarrow A = 0,$$

$$2\Big(A'' - iA'(Ez - \epsilon) + A\Big(c^2 + E\left(Ez^2 - i\right) - 2Ez\epsilon - M^2 + \epsilon^2\Big)\Big) = 0 \quad \Rightarrow 0 = 0,$$

$$f_{44}{}'' + f_{44}\left(c^2 + E\left(Ez^2 - i\right) - 2Ez\epsilon - M^2 + \epsilon^2\right) = 0,$$

$$f_{33}\left(c^2 + E\left(Ez^2 + i\right) - 2Ez\epsilon - M^2 + \epsilon^2\right) + f_{33}{}'' = 0,$$

$$f_{22}{}'' + f_{22}\left(c^2 + E\left(Ez^2 + i\right) - 2Ez\epsilon - M^2 + \epsilon^2\right) = 0,$$

$$f_{11}\left(c^2 + E\left(Ez^2 - i\right) - 2Ez\epsilon - M^2 + \epsilon^2\right) + f_{11}{}'' = 0.$$

Thus, we arrive at equations

$$D(z) = 0, \quad A(z) = 0,$$

$$\begin{aligned}
&\frac{d^2}{dz^2} f_{11} + f_{11}\left(E^2z^2 - iE - 2Ez\epsilon + \epsilon^2 - M^2 + c^2\right) = 0,\\
&\frac{d^2}{dz^2} f_{22} + f_{22}\left(E^2z^2 + iE - 2Ez\epsilon + \epsilon^2 - M^2 + c^2\right) = 0,\\
&\frac{d^2}{dz^2} f_{33} + f_{33}\left(E^2z^2 + iE - 2Ez\epsilon + \epsilon^2 - M^2 + c^2\right) = 0,\\
&\frac{d^2}{dz^2} f_{44} + f_{44}\left(E^2z^2 - iE - 2Ez\epsilon + \epsilon^2 - M^2 + c^2\right) = 0,
\end{aligned} \tag{28}$$

physical dimensions of involved parameters are $[z] = L, \quad [c] = [E] = \frac{1}{L^2}, \quad [Ez] = [\epsilon] = [M] = \frac{1}{L}.$

The last four equations may be re-written differently

$$\begin{aligned}
&\frac{d^2}{dz^2} f_{11} + ((\epsilon - Ez)^2 - iE - M^2 + c^2)\, f_{11} = 0,\\
&\frac{d^2}{dz^2} f_{44} + ((\epsilon - Ez)^2 - iE - M^2 + c^2)\, f_{44} = 0,\\
&\frac{d^2}{dz^2} f_{22} + ((\epsilon - Ez)^2 + iE - M^2 + c^2)\, f_{22} = 0,\\
&\frac{d^2}{dz^2} f_{33} + ((\epsilon - Ez)^2 + iE - M^2 + c^2)\, f_{33} = 0,
\end{aligned} \tag{29}$$

they may be solved in confluent hypergeometric functions So we have constructed four linearly independent solutions of the Dirac – Kähler particle. For each solution the corresponding remaining variables are

$$\underline{f_{11}(z)}, \qquad f_{12}(z)=0, \quad f_{21}=-\frac{f_{11}'+i(\epsilon-Ez)f_{11}}{c}, \quad f_{23}(z)=0, \quad f_{32}=0,$$

$$f_{14}(z)=0, \quad f_{41}=\frac{+iMf_{11}}{c}, \quad f_{34}(z)=0, \quad f_{43}=0,$$

$$\underline{f_{44}(z)}, \qquad f_{12}(z)=0, \quad f_{21}=0, \quad f_{23}(z)=0, f_{32}$$

$$f_{14}(z)=\frac{-iMf_{44}}{c}, \quad f_{41}=0, \quad f_{34}(z)=\frac{f_{44}'+i(\epsilon-Ez)f_{44}}{c}, \quad f_{43}=0,$$

$$\underline{f_{22}(z)}, \qquad f_{12}(z)=\frac{f_{22}'-i(\epsilon-Ez)f_{22}}{c}, \quad f_{21}=0, \quad f_{23}(z)=0, f_{32}$$

$$f_{14}(z)=0, \quad f_{41}=0, \quad f_{34}(z)=0, \quad f_{43}=0,$$

$$\underline{f_{33}(z)}, \qquad f_{12}(z)=0, \quad f_{21}=0, \quad f_{23}(z)=0, f_{32}$$

$$f_{14}(z)=0, \quad f_{41}=0, \quad f_{34}(z)=0, \quad f_{43}=-\frac{f_{33}'-i(\epsilon-Ez)f_{33}}{c}.$$

Conclusions

The system of equations describing the Dirac – Kähler particle in presence of the external electric field has been studied. After separating the variables in cylindrical coordinates, we derive the system of sixteen first order equations in partial derivative with respect to coordinates (r,z). To resolve this system, we apply the method by Fedorov – Gronskiy and decompose the complete wave function into the sue of three projective constituents. Accordingly, dependence of 16 variables on the polar coordinate is determined only through three basic functions $F_i(r)$, at this there arise differential constrains which permit to derive the system of 16 differential equations in the coordinate z. The last is solved exactly, as the result we have constructed four linearly independent solutions for the Dirac – Kähler particle in presence of the external uniform electric field.

REFERENCES

1. Dirac, P. A. M. The quantum theory of the electron / P. A. M. Dirac // Proc. Roy. Soc. A. – 1928. – Vol. 117. – P. 610–624; The quantum theory of the electron; Part II // Proc. Roy. Soc. A. – 1928. – Vol. 118. – P. 351–361.

2. Darwin, C. G. The electron as a vector wave / C. G. Darwin // Proc. Roy. Soc. Lond. A. – 1927. – Vol. 116. – P. 227–253; The wave equations of the electron // Proc. Roy. Soc. Lond. A. – 1928. – Vol. 118, nr 780. – P. 654–680.

3. Ivanenko, D. Zur theorie des magnetischen electrons / D. Ivanenko, L. Landau // Zeit. Phys. – 1928. – Vol. 48, nr 8. – P. 340–348.

4. Kähler, E. Der innere differentialkalkül / E. Kähler // Rendiconti di Mat. (Roma). – 1962. – Vol. 21, nr 3, 4. – P. 425–523.

5. Red'kov, V. M. On equations for the Dirac – Kähler field and bosons with different parities in the Riemannian space / V. M. Red'kov // Proceedings of the National Academy of Sciences of Belarus. Ser. fiz.-mat. – 2000. – № 1. – P. 90–95.

6. Red'kov, V. M. Dirac – Kähler field, spinor technique, and 2-potential approach to electrodynamics with two charges / V. M. Red'kov // Nonlinear Dynamics and Applications. – 2008. – Vol. 15. – P. 147–163.

7. Red'kov, V. M. Field particles in Riemannian space and the Lorentz group / V. M. Red'kov. – Minsk : Belarussian Science, 2009. – 486 p.

8. Red'kov, V. M. Tetrad formalism, spherical symmetry and Schrödinger basis group / V. M. Red'kov. – Minsk : Belarussian Science, 2011. – 339 p.

9. Red'kov, V. M. Dirac – Kähler field and 2-potential approach to electrodynamics with two charges. Chapter I / V. M. Red'kov // Progress in Relativity, Gravitation, Cosmology ; ed.: V. V. Dvoeglazov, A. Molgado, Universidad de Zacatecas, Mexico. – New York : Nova Science Publishers Inc., 2012. – P. 23–37.

10. Ovsiyuk, E. M. Some Consequences from the Dirac-Kähler Theory: on Intrinsic Spinor Sub-structure of the Different Boson Wave Functions / E. Ovsiyuk, O. Veko, V. Red'kov // Nonlinear Phenomena in Complex Systems. – 2013. – Vol. 16, nr 1. – P. 13–23.

11. Ovsiyuk, E. M. Maxwell Electrodynamics and Boson Fields in Spaces of Constant Curvature / E. M. Ovsiyuk, V. V. Kisel, V. M. Red'kov. – New York : Nova Science Publishers Inc., 2014. – 486 p.

12. Veko, O. V. Lounesto classification and the theory of the Dirac-Kähler field: on intrinsic spinor sub-structure of the different boson wave functions / O. V. Veko, E. M. Ovsiyuk, V. M. Red'kov // Relativity, Gravitation, Cosmology: Foundations ; ed. V. V. Dvoeglazov. – New York : Nova Science Publishers Inc., 2015. – P. 75–88.

13. Dirac – Kähler particle in Riemann spherical space: boson interpretation / A. M. Ishkhanyan [et al.] // Canad. J. Phys. – 2015. – Vol. 93. – P. 1427–1433.

14. The Dirac – Kähler field in spherical Riemann space: boson interpretation, exact solutions / A. M. Ishkhanyan, O. Florea, E. M. Ovsiyuk, V. M. Red'kov // Vesnik of Brest University. Series 4, Physics. Mathematics. – 2015. – № l. – P. 15–26.

15. Ovsiyuk, E. M. The Dirac – Kähler particle in Lobachevsky space, nonrelativistic approximation, exact solutions / E. M. Ovsiyuk, A. N. Red'ko, V. M. Red'kov // Proceedings of the National Academy of Sciences of Belarus. Ser. fiz.-mat. – 2015. – Vol. 4. – P. 61–70.

16. Chichurin, A. V. Dirac – Kähler particle in Riemann spherical space: analytical and numerical study, visualization / A. V. Chichurin, E. M. Ovsiyuk, V. M. Red'kov // Studia i Materialy. – 2015. – Vol. 1, nr 9. – P. 41–54.

17. Nonrelativistic description of the Dirac – Kähler particle on the background of curved space-time / K. V. Kazmerchuk, V. V. Kisel, E. M. Ovsiyuk [et al.] // Relativity, Gravitation, Cosmology: Foundations. – New York : Nova Science Publishers Inc., 2015. – P. 59–74.

18. Veko, O. V. Lounesto classification and the theory of the Dirac – Kähler field on intrinsic spinor sub-structure of the different boson wave functions / O. V. Veko, E. M. Ovsiyuk, V. M. Red'kov // Relativity, Gravitation, Cosmology: Foundations. – New York : Nova Science Publishers Inc., 2015. – P. 75–88.

19. Pletukhov, V. A. Relativistic wave equations and intrinsic degrees of freedom / V. A. Pletukhov, V. M. Red'kov, V. I. Strazhev. – Minsk : Belarussian Science, 2015. – 328 p.

20. Elementary Particles with Internal Structure in External Fields. Vol I. General Theory / V.V. Kisel [et al.] – New York : Nova Science Publishers Inc., 2018. – 404 p.

21. Elementary Particles with Internal Structure in External Fields. Vol II. Physical Problems / V. V. Kisel [et al.] – New York : Nova Science Publishers Inc., 2018. – 402 p.

22. Gronskiy, V. K. Magnetic properties of a particle with spin 3/2 / V. K. Gronskiy, F. I. Fedorov // Doklady of the National Academy of Sciences of Belarus. – 1960. – Vol. 4, № 7. – P. 278–283.

Рукапіс паступіў у рэдакцыю 20.05.2025