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***Vasily Kisel*[1]*, Anton Bury*[2]*, Polina Sachenok*[3]*, Elena Ovsiyuk*[4]**
[1]*Candidate of Physical and Mathematical Sciences, Associate Professor,*
Associate Professor of the Department of Natural Sciences
of Belarusian State Agrarian Technical University
[2]*3-d Postgraduate Student of Fundamental Interaction and Astrophysics Center*
of B. I. Stepanov Institute of Physics of National Academy of Sciences of Belarus
[3]*3-d Student of the Physics and Engineering Faculty*
of Mozyr State Pedagogical University named after I. P. Shamyakin
[4]*Doctor of Physical and Mathematical Sciences, Associate Professor,*
Leading Researcher of F. Skorina Gomel State University
Василий Васильевич Кисель*[1]*, Антон Васильевич Бурый*[2]*,
***Полина Олеговна Саченок*[3]*, Елена Михайловна Овсиюк*[4]**
[1]*канд. физ.-мат. наук, доц., доц. каф. естественнонаучных дисциплин*
Белорусского государственного аграрного технического университета
[2]*аспирант 3-го года обучения Центра фундаментальных взаимодействий и астрофизики*
Института физики имени Б. И. Степанова Национальной академии наук Беларуси
[3]*студент 3-го курса физико-инженерного факультета*
Мозырского государственного педагогического университета имени И. П. Шамякина
[4]*д-р физ.-мат. наук, доц., ведущий научный сотрудник*
Гомельского государственного университета имени Ф. Скорины
e-mail: [1]vasiliy_bspu@ mail.ru; [2]anton.buryy.97@mail.ru;
[3]polinasacenok@gmail.com; [4]e.ovsiyuk@mail.ru

SPIN 1/2 PARTICLE WITH ANOMALOUS MAGNETIC MOMENT AND POLARIZABILITY

Within the general method by Gel'fand – Yaglom, starting with the extended set of representations of the Lorentz group, we construct a relativistic generalized equation for a spin 1/2 particle with two additional characteristics. We take into account the presence of external electromagnetic fields. After eliminating the accessory variables of the complete wave function, we derive the generalized Dirac-like equation, the last includes two additional interaction terms which are interpreted as related to anomalous magnetic moment, and a second additional characteristics looks as related to polarizability of the particle.

Key words: *spin 1/2 particle, external electromagnetic fields, anomalous magnetic moment, polarizability.*

Частица со спином 1/2 с аномальным магнитным моментом и поляризуемостью

В рамках общего метода Гельфанда – Яглома, исходя из расширенного набора представлений группы Лоренца, строится релятивистское обобщенное уравнение для частицы со спином 1/2 с двумя дополнительными характеристиками. При этом учитывается наличие внешних электромагнитных полей. После исключения дополнительных переменных полной волновой функции получено обобщенное уравнение типа Дирака, включающее два дополнительных члена взаимодействия, один из которых связан с аномальным магнитным моментом, а второй – с поляризуемостью частицы.

Ключевые слова: *частица со спином 1/2, внешние электромагнитные поля, аномальный магнитный момент, поляризуемость.*

Introduction

Within the general method by Gel'fand – Yaglom [1−3], starting with the extended set of representations of the Lorentz group, we construct a relativistic generalized equation for a spin 1/2 particle with two additional characteristics. In [4], it was studied the same set of irreducible representations of the Lorentz group, but author restricted themselves only to a free particle theory. In the present paper, we have taken into account the external electromagnetic fields. After eliminating the accessory variables of the complete wave function, we derive the generalized Dirac-like equation, the last includes two additional interaction terms which are interpreted as related to anomalous magnetic moment [5−8], and a second additional characteristics which seems to be related to polarizability of the particle.

1. Gel'fand – Yaglom formalism

We will construct a P-invariant relativistic first order equation for a particle with the M and the spin $S = 1/2$

$$(\Gamma_\mu \partial_\mu + M)\Psi = 0 \qquad (1)$$

by using the set of representations of the Lorenz group with the following linking scheme

$$\begin{array}{ccc} 3(1/2,0) & - & 3(0,1/2) \\ | & & | \\ (1,1/2) & - & (1/2,1) \end{array} .$$

The corresponding system of spinor equations has the form

$$\partial^{\dot{a}b}(\lambda_1\psi_b + \lambda_2\varphi_b + \lambda_3\Phi_b) + \lambda_4\partial^b_{\dot{c}} f_b^{(\dot{a}\dot{c})} + M\psi^{\dot{a}} = 0, \qquad (2)$$

$$\partial_{a\dot{b}}(\lambda_1\psi^{\dot{b}} + \lambda_2\varphi^{\dot{b}} + \lambda_3\Phi^{\dot{b}}) + \lambda_4\partial^c_{\dot{b}} f^{\dot{b}}_{(ac)} + M\psi_a = 0, \qquad (3)$$

$$\partial^{\dot{a}b}(\lambda_5\psi_b + \lambda_6\varphi_b + \lambda_7\Phi_b) + \lambda_8\partial^b_{\dot{c}} f_b^{(\dot{a}\dot{c})} + M\varphi^{\dot{a}} = 0, \qquad (4)$$

$$\partial_{a\dot{b}}(\lambda_5\psi^{\dot{b}} + \lambda_6\varphi^{\dot{b}} + \lambda_7\Phi^{\dot{b}}) + \lambda_8\partial^c_{\dot{b}} f^{\dot{b}}_{(ac)} + M\varphi_a = 0, \qquad (5)$$

$$\partial^{\dot{a}b}(\lambda_9\psi_b + \lambda_{10}\varphi_b + \lambda_{11}\Phi_b) + \lambda_{12}\partial^b_{\dot{c}} f_b^{(\dot{a}\dot{c})} + M\Phi^{\dot{a}} = 0, \qquad (6)$$

$$\partial_{a\dot{b}}(\lambda_9\psi^{\dot{b}} + \lambda_{10}\varphi^{\dot{b}} + \lambda_{11}\Phi^{\dot{b}}) + \lambda_{12}\partial^c_{\dot{b}} f^{\dot{b}}_{(ac)} + M\Phi_a = 0, \qquad (7)$$

$$\frac{1}{2}[\partial^{\dot{a}}_c(\lambda_{13}\psi^{\dot{b}} + \lambda_{14}\varphi^{\dot{b}} + \lambda_{15}\Phi^{\dot{b}}) + \partial^{\dot{b}}_c(\lambda_{13}\psi^{\dot{a}} + \lambda_{14}\varphi^{\dot{a}} + \lambda_{15}\Phi^{\dot{a}})] +$$
$$+ \frac{\lambda_{16}}{2}[\partial^{k\dot{a}} f^{\dot{b}}_{(kc)} + \partial^{k\dot{b}} f^{\dot{a}}_{(kc)}] + Mf_c^{(\dot{a}\dot{b})} = 0, \qquad (8)$$

$$\frac{1}{2}[\partial^{\dot{c}}_a(\lambda_{13}\psi_b + \lambda_{14}\varphi_b + \lambda_{15}\Phi_b) + \partial^{\dot{c}}_b(\lambda_{13}\psi_a + \lambda_{14}\varphi_a + \lambda_{15}\Phi_a)] +$$
$$+ \frac{\lambda_{16}}{2}[\partial_{\dot{k}a} f_b^{(\dot{k}\dot{c})} + \partial_{\dot{k}b} f_a^{(\dot{k}\dot{c})}] + Mf^{\dot{c}}_{(ab)} = 0. \qquad (9)$$

The numerical parameter λ_i will be restricted later on. Below we will use the notations (we will apply the imaginery time coordinate $x_4 = ict$)

$$\partial_{\dot{a}b} = \frac{1}{i}\partial_\mu(\sigma^\mu)_{\dot{a}b}, \quad (\sigma^1)_{\dot{a}b} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}, \quad (\sigma^2)_{\dot{a}b} = \begin{vmatrix} 0 & -i \\ i & 0 \end{vmatrix}, \quad (\sigma^3)_{\dot{a}b} = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}, \quad (\sigma^4)_{\dot{a}b} = \begin{vmatrix} i & 0 \\ 0 & i \end{vmatrix}.$$

The operation of P-reflection is determined by the relations

$$\psi^{\dot{a}} \leftrightarrow \psi_a, \quad \varphi^{\dot{a}} \leftrightarrow \varphi_a, \quad \Phi^{\dot{a}} \leftrightarrow \Phi_a, \quad f_c^{(\dot{a}\dot{b})} \leftrightarrow f^{\dot{c}}_{ab}.$$

The system (2) – (9) can be presented in the matrix form). The components of the complete wave function ψ will be listed as follows

$$\psi^{(spinor)} = (\psi^{\dot{1}}, \psi^{\dot{2}}, \psi_1, \psi_2; \varphi^{\dot{1}}, \varphi^{\dot{2}}, \varphi_1, \varphi_2; \Phi^{\dot{1}}, \Phi^{\dot{2}}, \Phi_1, \Phi_2;$$

$$f^{\dot{1}}_{(11)}, f^{\dot{1}}_{(12)}, f^{\dot{1}}_{(22)}, f^{\dot{2}}_{(11)}, f^{\dot{2}}_{(12)}, f^{\dot{2}}_{(22)}, f_1^{(\dot{1}\dot{1})}, f_1^{(\dot{1}\dot{2})}, f_1^{(\dot{2}\dot{2})}, f_2^{(\dot{1}\dot{1})}, f_2^{(\dot{1}\dot{2})}, f_2^{(\dot{2}\dot{2})})_T;$$

the symbol T stands for the matrix transposition. Correspondingly, the matrix Γ_4^{spinor} has the form

$$\Gamma_4^{spinor} = \left| \begin{array}{cccccccccccc}
0 & 0 & \lambda_1 & 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & \lambda_3 & 0 \\
0 & 0 & 0 & \lambda_1 & 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & \lambda_3 \\
\lambda_1 & 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & \lambda_3 & 0 & 0 & 0 \\
0 & \lambda_1 & 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & \lambda_3 & 0 & 0 \\
0 & 0 & \lambda_5 & 0 & 0 & 0 & \lambda_6 & 0 & 0 & 0 & \lambda_7 & 0 \\
0 & 0 & 0 & \lambda_5 & 0 & 0 & 0 & \lambda_6 & 0 & 0 & 0 & \lambda_7 \\
\lambda_5 & 0 & 0 & 0 & \lambda_6 & 0 & 0 & 0 & \lambda_7 & 0 & 0 & 0 \\
0 & \lambda_5 & 0 & 0 & 0 & \lambda_6 & 0 & 0 & 0 & \lambda_7 & 0 & 0 \\
0 & 0 & \lambda_9 & 0 & 0 & 0 & \lambda_{10} & 0 & 0 & 0 & \lambda_{11} & 0 \\
0 & 0 & 0 & \lambda_9 & 0 & 0 & 0 & \lambda_{10} & 0 & 0 & 0 & \lambda_{11} \\
\lambda_9 & 0 & 0 & 0 & \lambda_{10} & 0 & 0 & 0 & \lambda_{11} & 0 & 0 & 0 \\
0 & \lambda_9 & 0 & 0 & 0 & \lambda_{10} & 0 & 0 & 0 & \lambda_{11} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\lambda_{13}}{2} & 0 & 0 & 0 & \frac{\lambda_{14}}{2} & 0 & 0 & 0 & \frac{\lambda_{15}}{2} & 0 \\
0 & 0 & 0 & \lambda_{13} & 0 & 0 & 0 & \lambda_{14} & 0 & 0 & 0 & \lambda_{15} \\
0 & 0 & -\lambda_{13} & 0 & 0 & 0 & -\lambda_{14} & 0 & 0 & 0 & -\lambda_{15} & 0 \\
0 & 0 & 0 & -\frac{\lambda_{13}}{2} & 0 & 0 & 0 & -\frac{\lambda_{14}}{2} & 0 & 0 & 0 & -\frac{\lambda_{15}}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{\lambda_{13}}{2} & 0 & 0 & 0 & -\frac{\lambda_{14}}{2} & 0 & 0 & 0 & -\frac{\lambda_{15}}{2} & 0 & 0 & 0 \\
0 & -\lambda_{13} & 0 & 0 & 0 & -\lambda_{14} & 0 & 0 & 0 & -\lambda_{15} & 0 & 0 \\
\lambda_{13} & 0 & 0 & 0 & \lambda_{14} & 0 & 0 & 0 & \lambda_{15} & 0 & 0 & 0 \\
0 & \frac{\lambda_{13}}{2} & 0 & 0 & 0 & \frac{\lambda_{14}}{2} & 0 & 0 & 0 & \frac{\lambda_{15}}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array} \right.$$

$$
\left.\begin{array}{cccccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_4 & 0 & -\lambda_4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_4 & 0 & -\lambda_4 & 0 \\
0 & -\lambda_4 & 0 & \lambda_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\lambda_4 & 0 & \lambda_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_8 & 0 & -\lambda_8 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_8 & 0 & -\lambda_8 & 0 \\
0 & -\lambda_8 & 0 & \lambda_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\lambda_8 & 0 & \lambda_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{12} & 0 & -\lambda_{12} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{12} & 0 & -\lambda_{12} & 0 \\
0 & -\lambda_{12} & 0 & \lambda_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\lambda_{12} & 0 & \lambda_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \lambda_{16} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\lambda_{16}}{2} & 0 & \frac{\lambda_{16}}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{16} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{16} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\lambda_{16}}{2} & 0 & \frac{\lambda_{16}}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{16} \\
\lambda_{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\lambda_{16}}{2} & 0 & \frac{\lambda_{16}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \lambda_{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \lambda_{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\lambda_{16}}{2} & 0 & \frac{\lambda_{16}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \lambda_{16} & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right| . \qquad (10)
$$

Let us transform the matrix Γ_4 to the modified Gelfand – Yaglom basis (10) [3]; the last is more convenient for obtaining restrictions on parameters λ_i.

To this end, first we find the matrix Γ^4 in canonic basis.

For the use set of representations of the Lorenz group $SO(3,1)$, the wave function in canonical basis has the structure

$$
\psi^{canon} = (\psi^{(1/2,0)}_{(1/2,0)}, \psi^{(1/2,0)}_{(-1/2,0)}, \psi^{(0,1/2)}_{(0,1/2)}, \psi^{(0,1/2)}_{(0,-1/2)}; \varphi^{(1/2,0)}_{(1/2,0)}, \varphi^{(1/2,0)}_{(-1/2,0)}, \varphi^{(0,1/2)}_{(0,1/2)}, \varphi^{(0,1/2)}_{(0,-1/2)};
$$

$$
\Phi^{(1/2,0)}_{(1/2,0)}, \Phi^{(1/2,0)}_{(-1/2,0)}, \Phi^{(0,1/2)}_{(0,1/2)}, \Phi^{(0,1/2)}_{(0,-1/2)}; f^{(1/2,1)}_{(1/2,1)}, f^{(1/2,1)}_{(-1/2,1)}, f^{(1/2,1)}_{(1/2,0)}, f^{(1/2,1)}_{(-1/2,0)}, f^{(1/2,1)}_{(1/2,-1)}, f^{(1/2,1)}_{(-1/2,-1)};
$$

$$
f^{(1,1/2)}_{(1,1/2)}, f^{(1,1/2)}_{(0,1/2)}, f^{(1,1/2)}_{(-1,1/2)}, f^{(1,1/2)}_{(1,-1/2)}, f^{(1,1/2)}_{(0,-1/2)}, f^{(1,1/2)}_{(-1,-1/2)})_T .
$$

Functions in two presentations are connected by transformation $\psi^{canon} = B\psi^{spinor}$, its general structure is determined by the formula

$$\psi^{(l,l')}_{(l_3,l_{3'})} = [\frac{(2l)!}{(l+l_3)!(l-l_3)!}]^{1/2}[\frac{(2l')!}{(l'+l_{3'})!(l'+l_{3'})!}]^{1/2}\psi^{(\dot{1}\dot{1}\dot{1}\cdots\dot{1}\dot{2}\cdots\dot{2})}_{(1\cdots 12\cdots 2)},$$

where parameters $(l_3, l_{3'})$ define the functions, transformed as irreducible representation (l,l') for the proper Lorenz group; the number of spinor indices $\dot{1}$ is equal to $(l+l_3)$, the number of spinor indices $\dot{2}$ is equal to $(l-l_3)$; the number of spinor indices 1 is equal to $(l'+l_{3'})$, 2 – $(l'-l_{3'})$. Correspondingly, the matrix Γ_4 transforming by the formulas: $\Gamma_4^{canon} = B\Gamma_4^{spinor}B^{-1}$. Explicitly the matrix B reads

$$B = \left|\begin{array}{ccccccccccccccc}
I_4 & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & I_4 & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & I_4 & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & 1 & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & 1 & . & . & . & . & . & . & . & . \\
. & . & . & . & \sqrt{2} & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & \sqrt{2} & . & . & . & . & . & . & . \\
. & . & . & . & . & 1 & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & 1 & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & 1 & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & \sqrt{2} & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & 1 & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & 1 & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & \sqrt{2} & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & 1
\end{array}\right|.$$

Further we get

$$\Gamma_4^{canon} = \left| \begin{matrix}
0 & 0 & \lambda_1 & 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & \lambda_3 & 0 \\
0 & 0 & 0 & \lambda_1 & 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & \lambda_3 \\
\lambda_1 & 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & \lambda_3 & 0 & 0 & 0 \\
0 & \lambda_1 & 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & \lambda_3 & 0 & 0 \\
0 & 0 & \lambda_5 & 0 & 0 & 0 & \lambda_6 & 0 & 0 & 0 & \lambda_7 & 0 \\
0 & 0 & 0 & \lambda_5 & 0 & 0 & 0 & \lambda_6 & 0 & 0 & 0 & \lambda_7 \\
\lambda_5 & 0 & 0 & 0 & \lambda_6 & 0 & 0 & 0 & \lambda_7 & 0 & 0 & 0 \\
0 & \lambda_5 & 0 & 0 & 0 & \lambda_6 & 0 & 0 & 0 & \lambda_7 & 0 & 0 \\
0 & 0 & \lambda_9 & 0 & 0 & 0 & \lambda_{10} & 0 & 0 & 0 & \lambda_{11} & 0 \\
0 & 0 & 0 & \lambda_9 & 0 & 0 & 0 & \lambda_{10} & 0 & 0 & 0 & \lambda_{11} \\
\lambda_9 & 0 & 0 & 0 & \lambda_{10} & 0 & 0 & 0 & \lambda_{11} & 0 & 0 & 0 \\
0 & \lambda_9 & 0 & 0 & 0 & \lambda_{10} & 0 & 0 & 0 & \lambda_{11} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\lambda_{13} & 0 & 0 & 0 & -\lambda_{14} & 0 & 0 & 0 & -\lambda_{15} & 0 \\
0 & 0 & \frac{\lambda_{13}}{\sqrt{2}} & 0 & 0 & 0 & \frac{\lambda_{14}}{\sqrt{2}} & 0 & 0 & 0 & \frac{\lambda_{15}}{\sqrt{2}} & 0 \\
0 & 0 & 0 & -\frac{\lambda_{13}}{\sqrt{2}} & 0 & 0 & 0 & -\frac{\lambda_{14}}{\sqrt{2}} & 0 & 0 & 0 & -\frac{\lambda_{15}}{\sqrt{2}} \\
0 & 0 & 0 & \lambda_{13} & 0 & 0 & 0 & \lambda_{14} & 0 & 0 & 0 & \lambda_{15} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{\lambda_{13}}{\sqrt{2}} & 0 & 0 & 0 & -\frac{\lambda_{14}}{\sqrt{2}} & 0 & 0 & 0 & -\frac{\lambda_{15}}{\sqrt{2}} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\lambda_{16}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\lambda_{16}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\lambda_{16}}{2} & \frac{\lambda_{16}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \lambda_{16} & 0 & 0 & 0 & 0 & 0 & 0
\end{matrix} \right.$$

$$
\left.\begin{array}{cccccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\lambda_4}{\sqrt{2}} & 0 & -\lambda_4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_4 & 0 & -\frac{\lambda_4}{\sqrt{2}} & 0 \\
0 & \lambda_4 & -\frac{\lambda_4}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\lambda_4}{\sqrt{2}} & -\lambda_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\lambda_8}{\sqrt{2}} & 0 & -\lambda_8 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_8 & 0 & -\frac{\lambda_8}{\sqrt{2}} & 0 \\
0 & \lambda_8 & -\frac{\lambda_8}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{\lambda_8}{\sqrt{2}} & -\lambda_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\lambda_{12}}{\sqrt{2}} & 0 & -\lambda_{12} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{12} & 0 & -\frac{\lambda_{12}}{\sqrt{2}} & 0 \\
0 & \lambda_{12} & -\frac{\lambda_{12}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\lambda_{12}}{\sqrt{2}} & -\lambda_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \lambda_{16} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\lambda_{16}}{\sqrt{2}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\lambda_{16}}{2} & 0 & \frac{\lambda_{16}}{\sqrt{2}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\lambda_{16}}{\sqrt{2}} & 0 & \frac{\lambda_{16}}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{16}/2 & 0 & \lambda_{16}/\sqrt{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{16} \\
\lambda_{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\lambda_{16}}{\sqrt{2}} & \frac{\lambda_{16}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\lambda_{16}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\lambda_{16}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\lambda_{16}}{2} & \frac{\lambda_{16}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \lambda_{16} & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right| .
$$

Let us find the matrix Γ_4 in modified Gelfand – Yaglom basis:

$$\psi^{G-Y} = A\psi^{canon};$$

$$\psi^{G-Y} = (\psi^{(1/2,0)}_{(1/2,1/2)}, \psi^{(1/2,0)}_{(1/2,-1/2)}, \psi^{(0,1/2)}_{(1/2,1/2)}, \psi^{(0,1/2)}_{(1/2,-1/2)}; \varphi^{(1/2,0)}_{(1/2,1/2)}, \varphi^{(1/2,0)}_{(1/2,-1/2)}, \varphi^{(0,1/2)}_{(1/2,1/2)}, \varphi^{(0,1/2)}_{(1/2,-1/2)};$$

$$\Phi^{(1/2,0)}_{(1/2,1/2)}, \Phi^{(1/2,0)}_{(1/2,-1/2)}, \Phi^{(0,1/2)}_{(1/2,1/2)}, \Phi^{(0,1/2)}_{(1/2,-1/2)}; f^{(1/2,1)}_{(1/2,1/2)}, f^{(1/2,1)}_{(1/2,-1/2)}, f^{(1,1/2)}_{(1/2,1/2)}, f^{(1,1/2)}_{(1/2,-1/2)}; f^{(1/2,1)}_{(3/2,3/2)}, f^{(1/2,1)}_{(3/2,-3/2)},$$

$$f^{(1,1/2)}_{(3/2,3/2)}, f^{(1,1/2)}_{(3/2,-3/2)}; f^{(1/2,1)}_{(3/2,1/2)}, f^{(1/2,1)}_{(3/2,-1/2)}, f^{(1,1/2)}_{(3/2,1/2)}, f^{(1,1/2)}_{(3/2,-1/2)})_T;$$

$$\Gamma_4^{G-Y} = A\Gamma_4^{canon}A^{-1};$$

the structure of the matrix A is governed by the formula

$$\psi^{(l,l')}_{s,m} = \sum C^{sm}_{ll_3,l'l_{3'}} \psi^{(l,l')}_{(l_3,l_{3'})}, \quad m = l_3 + l_{3'},$$

where $C^{sm}_{ll_3,l'l_{3'}}$ stand for Clebsch – Gordan coefficients; the summing is done over the parameters $l_3, l_{3'}$, related as $l_3 + l_{3'} = m$; in this way, we get

$$B = \left|\begin{array}{ccccccccccccccc}
I_4 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & I_4 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & I_4 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & -\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{\sqrt{3}} & \cdot & \sqrt{\frac{2}{3}} & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\sqrt{\frac{2}{3}} & \cdot & \frac{1}{\sqrt{3}} & \cdot \\
\cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 \\
\cdot & \cdot & \cdot & \cdot & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \sqrt{\frac{2}{3}} & \cdot & \frac{1}{\sqrt{3}} & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{\sqrt{3}} & \cdot & \sqrt{\frac{2}{3}} & \cdot
\end{array}\right|,$$

and

$$A^{-1}=A^{+}=\begin{vmatrix}
I_4 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & I_4 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & I_4 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & -\sqrt{\frac{2}{3}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{\sqrt{3}} & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \frac{1}{\sqrt{3}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \sqrt{\frac{2}{3}} & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & -\frac{1}{\sqrt{3}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \sqrt{\frac{2}{3}} & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \sqrt{\frac{2}{3}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{\sqrt{3}} & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{\sqrt{3}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \sqrt{\frac{2}{3}} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\sqrt{\frac{2}{3}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{\sqrt{3}} \\
\cdot & \cdot & \cdot & \cdot & \cdot & \sqrt{\frac{2}{3}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{\sqrt{3}} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{\sqrt{3}} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \sqrt{\frac{2}{3}} \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot
\end{vmatrix}.$$

This results in

$$\Gamma_4^{G-Y} = \left| \begin{array}{cccccccccccc}
0 & 0 & \lambda_1 & 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & \lambda_3 & 0 \\
0 & 0 & 0 & \lambda_1 & 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & \lambda_3 \\
\lambda_1 & 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & \lambda_3 & 0 & 0 & 0 \\
0 & \lambda_1 & 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & \lambda_3 & 0 & 0 \\
0 & 0 & \lambda_5 & 0 & 0 & 0 & \lambda_6 & 0 & 0 & 0 & \lambda_7 & 0 \\
0 & 0 & 0 & \lambda_5 & 0 & 0 & 0 & \lambda_6 & 0 & 0 & 0 & \lambda_7 \\
\lambda_5 & 0 & 0 & 0 & \lambda_6 & 0 & 0 & 0 & \lambda_7 & 0 & 0 & 0 \\
0 & \lambda_5 & 0 & 0 & 0 & \lambda_6 & 0 & 0 & 0 & \lambda_7 & 0 & 0 \\
0 & 0 & \lambda_9 & 0 & 0 & 0 & \lambda_{10} & 0 & 0 & 0 & \lambda_{11} & 0 \\
0 & 0 & 0 & \lambda_9 & 0 & 0 & 0 & \lambda_{10} & 0 & 0 & 0 & \lambda_{11} \\
\lambda_9 & 0 & 0 & 0 & \lambda_{10} & 0 & 0 & 0 & \lambda_{11} & 0 & 0 & 0 \\
0 & \lambda_9 & 0 & 0 & 0 & \lambda_{10} & 0 & 0 & 0 & \lambda_{11} & 0 & 0 \\
0 & 0 & \sqrt{\frac{3}{2}}\lambda_{13} & 0 & 0 & 0 & \sqrt{\frac{3}{2}}\lambda_{14} & 0 & 0 & 0 & \sqrt{\frac{3}{2}}\lambda_{15} & 0 \\
0 & 0 & 0 & \sqrt{\frac{3}{2}}\lambda_{13} & 0 & 0 & 0 & \sqrt{\frac{3}{2}}\lambda_{14} & 0 & 0 & 0 & \sqrt{\frac{3}{2}}\lambda_{15} \\
\sqrt{\frac{3}{2}}\lambda_{13} & 0 & 0 & 0 & \sqrt{\frac{3}{2}}\lambda_{14} & 0 & 0 & 0 & \sqrt{\frac{3}{2}}\lambda_{15} & 0 & 0 & 0 \\
0 & \sqrt{\frac{3}{2}}\lambda_{13} & 0 & 0 & 0 & \sqrt{\frac{3}{2}}\lambda_{14} & 0 & 0 & 0 & \sqrt{\frac{3}{2}}\lambda_{15} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array} \right.$$

$$
\left.\begin{array}{cccccccccccc}
0 & 0 & -\sqrt{\frac{3}{2}}\lambda_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\sqrt{\frac{3}{2}}\lambda_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\sqrt{\frac{3}{2}}\lambda_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\sqrt{\frac{3}{2}}\lambda_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\sqrt{\frac{3}{2}}\lambda_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\sqrt{\frac{3}{2}}\lambda_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\sqrt{\frac{3}{2}}\lambda_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\sqrt{\frac{3}{2}}\lambda_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\sqrt{\frac{3}{2}}\lambda_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\sqrt{\frac{3}{2}}\lambda_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\sqrt{\frac{3}{2}}\lambda_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\sqrt{\frac{3}{2}}\lambda_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2}\lambda_{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2}\lambda_{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{2}\lambda_{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{2}\lambda_{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \lambda_{16} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{16} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \lambda_{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \lambda_{16} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{16} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{16} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{16} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{16} & 0 & 0
\end{array}\right| .
$$

This expression may be presented in a short form (which is the result of the use of the modified Gelfand – Yaglom basis):

$$\Gamma_4^{G-Y} = \begin{vmatrix} \beta^{(1/2)} \otimes \gamma_4 & 0 \\ 0 & \beta^{(3/2)} \otimes I_2 \otimes \gamma_4 \end{vmatrix},$$

where

$$\beta^{(1/2)} = \begin{vmatrix} \lambda_1 & \lambda_2 & \lambda_3 & -\sqrt{\frac{3}{2}}\lambda_4 \\ \lambda_5 & \lambda_6 & \lambda_7 & -\sqrt{\frac{3}{2}}\lambda_8 \\ \lambda_9 & \lambda_{10} & \lambda_{11} & -\sqrt{\frac{3}{2}}\lambda_{12} \\ \sqrt{\frac{3}{2}}\lambda_{13} & \sqrt{\frac{3}{2}}\lambda_{14} & \sqrt{\frac{3}{2}}\lambda_{15} & \frac{1}{2}\lambda_{16} \end{vmatrix}, \qquad \beta^{(3/2)} = \lambda_{16}, \quad \gamma_4 = \begin{vmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix}$$

We can readily verify that the P - invariant conditions are satisfied

$$\Gamma_4^{G-Y} P = P\Gamma_4^{G-Y}, P = \begin{vmatrix} P^{(1/2)} \otimes \gamma_4 & 0 \\ 0 & P^{(3/2)} \otimes I_2 \otimes \gamma_4 \end{vmatrix}, P^{(1/2)} = I_4, \quad P^{(3/2)} = -1.$$

2. The Lagrangian form of the theory

In modified Gelfand – Yaglom basis the matrix of the bilinear form has the structure

$$\eta = \begin{vmatrix} \eta^{(1/2)} \otimes \gamma_4 & 0 \\ 0 & \eta^{(3/2)} \otimes I_2 \otimes \gamma_4 \end{vmatrix}, \quad \eta^{(1/2)} = \begin{vmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{vmatrix}, \quad \eta^{(3/2)} = -k_4, k_i = \pm 1.$$

From the constraint $(\eta\Gamma_4)^+ = \eta\Gamma_4$ it follows

$$\lambda_1^* = \lambda_1, \quad \lambda_6^* = \lambda_6, \quad \lambda_{11}^* = \lambda_{11}, \quad \lambda_{16}^* = \lambda_{16},$$
$$\lambda_5^* = k_1 k_2 \lambda_2, \quad \lambda_9^* = k_1 k_3 \lambda_3, \quad \lambda_{10}^* = k_2 k_3 \lambda_7,$$
$$\lambda_{13}^* = -k_1 k_4 \lambda_4, \quad \lambda_{14}^* = -k_2 k_4 \lambda_8, \quad \lambda_{15}^* = -k_3 k_4 \lambda_{12}^*.$$

For instants, if $k_1 = +1, k_2 = -1, k_3 = -1, k_4 = -1$, then we have

$$\lambda_5^* = -\lambda_2, \quad \lambda_9^* = -\lambda_3, \quad \lambda_{10}^* = \lambda_7, \quad \lambda_{13}^* = \lambda_4, \quad \lambda_{14}^* = -\lambda_8, \quad \lambda_{15}^* = -\lambda_{12};$$

just this variant is considered below.

3. Restrictions on the parameters λ_i

Let us find restrictions on the parameters λ_i, followed from the requirement to have the theory for a particle with single mass state M and single spin $S = 1/2$ state; the last means that the state with spin $S = 3/2$ is absent

$$\lambda_{16} = 0.$$

Thus, the structure of spin blocks is

$$\beta^{(1/2)} = \begin{vmatrix} \lambda_1 & \lambda_2 & \lambda_3 & -\sqrt{\frac{3}{2}}\lambda_4 \\ \lambda_5 & \lambda_6 & \lambda_7 & -\sqrt{\frac{3}{2}}\lambda_8 \\ \lambda_9 & \lambda_{10} & \lambda_{11} & -\sqrt{\frac{3}{2}}\lambda_{12} \\ \sqrt{\frac{3}{2}}\lambda_{13} & \sqrt{\frac{3}{2}}\lambda_{14} & \sqrt{\frac{3}{2}}\lambda_{15} & 0 \end{vmatrix}, \quad \beta^{(3/2)} = 0. \tag{11}$$

Besides, because the matrix $\beta^{(1/2)}$ should have only one proper value $(+1)$, and all remaining ones should be equal to zero, We have additional constraints

$$\lambda_1 + \lambda_6 + \lambda_{11} = 1, \tag{12}$$

$$\lambda_2\lambda_5 + \lambda_3\lambda_9 + \lambda_7\lambda_{10} - \lambda_1\lambda_6 - \lambda_1\lambda_{11} - \lambda_6\lambda_{11} - \frac{3}{2}(\lambda_4\lambda_{13} + \lambda_8\lambda_{14} + \lambda_{12}\lambda_{15}) = 0, \tag{13}$$

$$\begin{gathered}(\lambda_1\lambda_6 + \lambda_1\lambda_{11} + \lambda_6\lambda_{11}) - (\lambda_1\lambda_6\lambda_{11} + \lambda_1\lambda_2\lambda_5 + \lambda_1\lambda_3\lambda_9 + \lambda_2\lambda_5\lambda_6 + \\ + \lambda_2\lambda_7\lambda_9 + \lambda_3\lambda_5\lambda_{10} + \lambda_3\lambda_9\lambda_{11} + \lambda_6\lambda_7\lambda_{10} + \lambda_7\lambda_{10}\lambda_{11}) + \\ + \frac{3}{2}(\lambda_1\lambda_4\lambda_{13} + \lambda_2\lambda_8\lambda_{13} + \lambda_3\lambda_{12}\lambda_{13} + \lambda_4\lambda_5\lambda_{14} + \lambda_4\lambda_9\lambda_{15} + \\ + \lambda_6\lambda_8\lambda_{14} + \lambda_7\lambda_{12}\lambda_{14} + \lambda_8\lambda_{10}\lambda_{15} + \lambda_{11}\lambda_{12}\lambda_{15}) = 0,\end{gathered} \tag{14}$$

$$\begin{gathered}\lambda_{13}(\lambda_2\lambda_7\lambda_{12} + \lambda_4\lambda_6\lambda_{11} + \lambda_3\lambda_8\lambda_{10} - \lambda_4\lambda_7\lambda_{10} - \lambda_3\lambda_6\lambda_{12} - \lambda_2\lambda_8\lambda_{11}) + \\ + \lambda_{14}(\lambda_4\lambda_7\lambda_9 + \lambda_3\lambda_5\lambda_{12} + \lambda_1\lambda_8\lambda_{11} - \lambda_1\lambda_7\lambda_{12} - \lambda_4\lambda_5\lambda_{11} - \lambda_3\lambda_8\lambda_9) + \\ + \lambda_{15}(\lambda_1\lambda_6\lambda_{12} + \lambda_4\lambda_5\lambda_{10} + \lambda_2\lambda_8\lambda_9 - \lambda_4\lambda_6\lambda_9 - \lambda_2\lambda_5\lambda_{12} - \lambda_1\lambda_8\lambda_{10}) = 0.\end{gathered} \tag{15}$$

The last are results of two equations

$$Sp(\beta^{(1/2)})^n = 1, \quad \det\beta^{(1/2)} = 0.$$

The relations (12) – (15) looks cumbersome. They may be simplify, if we take into account the possibility to break some links between repeated representations of the Lorenz group. At this, the physical content of the results remains the same. Such a break is reached by imposing the following conditions

$$\lambda_2 = \lambda_3 = \lambda_5 = \lambda_7 = \lambda_9 = \lambda_{10} = 0. \tag{16}$$

Correspondingly, we obtain

$$\lambda_1 + \lambda_6 + \lambda_{11} = 1, \tag{17}$$

$$\lambda_1\lambda_6 + \lambda_1\lambda_{11} + \lambda_6\lambda_{11} + \frac{3}{2}(\lambda_4\lambda_{13} + \lambda_8\lambda_{14} + \lambda_{12}\lambda_{15}) = 0, \tag{18}$$

$$(\lambda_1\lambda_6 + \lambda_1\lambda_{11} + \lambda_6\lambda_{11}) - \lambda_1\lambda_6\lambda_{11} + \frac{3}{2}(\lambda_1\lambda_4\lambda_{13} + \lambda_6\lambda_8\lambda_{14} + \lambda_{11}\lambda_{12}\lambda_{15}) = 0, \tag{19}$$

$$\lambda_4\lambda_6\lambda_{11}\lambda_{13} + \lambda_1\lambda_8\lambda_{11}\lambda_{14} + \lambda_1\lambda_6\lambda_{12}\lambda_{15} = 0. \tag{20}$$

Taking in mind restrictions (12) – (15) or (16) – (20), we get the presentation for minimal polynomial of $\beta^{(1/2)}$ (11)

$$(\beta^{1/2})^3[\beta^{(1/2)}-1]=0;$$

the minimal polynomial for the matrix Γ_4 is $\Gamma_4^3(\Gamma_4^2-1)=0$.

4. Spin-tensor form of equations

For the following, it is convenient to transform equations to spin-tensor form. At this, we will take into account the known relationships

$$f_m^{(\dot{k}\dot{n})}=\delta_{(\dot{r}\dot{s})}^{(\dot{k}\dot{n})}(\sigma^\mu)_m^{\dot{r}} f_\mu^{\dot{s}},\quad f_{(kn)}^{\dot{m}}=\delta_{(kn)}^{(rs)}(\sigma^\mu)_r^{\dot{m}} f_{s\mu}, \tag{21}$$

where $\delta_{(\dot{r}\dot{s})}^{(\dot{k}\dot{n})},\delta_{(kn)}^{(rs)}$ are the generalized Kronecker spinorial symbols

$$\delta_{(\dot{r}\dot{s})}^{(\dot{k}\dot{n})}=\frac{1}{2}(\delta_{\dot{r}}^{\dot{k}}\delta_{\dot{s}}^{\dot{n}}+\delta_{\dot{s}}^{\dot{k}}\delta_{\dot{r}}^{\dot{n}}),\quad \delta_{(kn)}^{(rs)}=\frac{1}{2}(\delta_k^r\delta_n^s+\delta_n^r\delta_k^s).$$

Two first spinor equations may be presented as follows

$$\lambda_1\partial^{\dot{a}b}\psi_b+\frac{\lambda_4}{2}[\partial_{\dot{b}}^c(\sigma^\mu)_c^{\dot{a}} f_\mu^{\dot{b}}+\partial_{\dot{b}}^c(\sigma^\mu)_c^{\dot{b}} f_\mu^{\dot{a}}]+M\psi^{\dot{a}}=0,$$

$$\lambda_1\partial_{a\dot{b}}\psi^{\dot{b}}+\frac{\lambda_4}{2}[\partial_{\dot{c}}^b(\sigma^\mu)_a^{\dot{c}} f_{b\mu}+\partial_{\dot{c}}^b(\sigma^\mu)_b^{\dot{c}} f_{a\mu}]+M\psi_a=0,$$

or

$$\frac{\lambda_1}{i}\partial_\mu(\sigma^\mu)^{\dot{a}b}\psi_b+\frac{\lambda_4}{2i}\partial_\nu[-(\sigma^\mu)^{\dot{a}c}(\sigma^\nu)_{c\dot{b}} f_\mu^{\dot{b}}-(\sigma^\nu)^{\dot{b}c}(\sigma^\mu)_{c\dot{b}} f_\mu^{\dot{a}}]+M\psi^{\dot{a}}=0,$$

$$\frac{\lambda_1}{i}\partial_\mu(\sigma^\mu)_{a\dot{b}}\psi^{\dot{b}}+\frac{\lambda_4}{2i}\partial_\nu[-(\sigma^\mu)_{a\dot{c}}(\sigma^\nu)^{\dot{c}b} f_{b\mu}-(\sigma^\nu)_{b\dot{c}}(\sigma^\mu)^{\dot{c}b} f_{a\mu}]+M\psi_a=0.$$

Taking in mind the identity

$$(\sigma^\mu)^{\dot{a}b}(\sigma^\nu)_{b\dot{a}}=(\sigma^\mu)_{a\dot{b}}(\sigma^\nu)^{\dot{b}a}=-2\delta_{\mu\nu} \tag{22}$$

we get

$$\frac{\lambda_1}{i}\partial_\mu(\sigma^\mu)^{\dot{a}b}\psi_b+\frac{\lambda_4}{2i}\partial_\nu[-(\sigma^\mu)^{\dot{a}c}(\sigma^\nu)_{c\dot{b}} f_\mu^{\dot{b}}+2f_\mu^{\dot{a}}]+M\psi^{\dot{a}}=0,$$

$$\frac{\lambda_1}{i}\partial_\mu(\sigma^\mu)_{a\dot{b}}\psi^{\dot{b}}+\frac{\lambda_4}{2i}\partial_\nu[-(\sigma^\mu)_{a\dot{c}}(\sigma^\nu)^{\dot{c}b} f_{b\mu}+2f_{a\mu}]+M\psi_a=0.$$

Two last equations may be joined to the one

$$\frac{\lambda_1}{i}\partial_\mu\begin{vmatrix}0 & (\sigma^\mu)^{\dot{a}b}\\ (\sigma^\mu)_{\dot{a}b} & 0\end{vmatrix}\begin{vmatrix}\psi^{\dot{b}}\\ \psi_b\end{vmatrix}+$$

$$+\frac{\lambda_4}{2i}\partial_\nu\begin{vmatrix}-(\sigma^\mu)^{\dot{a}c}(\sigma^\nu)_{c\dot{b}}+2\delta_{\dot{b}}^{\dot{a}}\delta_{\nu\mu} & 0\\ 0 & -(\sigma^\mu)_{\dot{a}c}(\sigma^\nu)^{c\dot{b}}+2\delta_b^a\delta_{\nu\mu}\end{vmatrix}\begin{vmatrix}f_\mu^{\dot{b}}\\ f_{b\mu}\end{vmatrix}+M\begin{vmatrix}\psi^{\dot{a}}\\ \psi_a\end{vmatrix}=0.$$

Due to identities

$$(\sigma^{\mu})^{\dot{a}b}(\sigma^{\nu})_{b\dot{c}}+(\sigma^{\nu})^{\dot{a}b}(\sigma^{\mu})_{b\dot{c}}=-2\delta_{\mu\nu}\delta^{\dot{a}}_{\dot{c}},\quad (\sigma^{\mu})_{\dot{a}b}(\sigma^{\nu})^{b\dot{c}}+(\sigma^{\nu})_{\dot{a}b}(\sigma^{\mu})^{b\dot{c}}=-2\delta_{\mu\nu}\delta^{c}_{a}, \tag{23}$$

$$\gamma_{\mu}=\frac{1}{i}\begin{vmatrix}0 & (\sigma^{\mu})^{\dot{a}b}\\ (\sigma^{\mu})_{\dot{a}b} & 0\end{vmatrix},$$

the previous equation is written as follows

$$\lambda_1\hat{\partial}\psi+\frac{\lambda_4}{2i}\partial_{\nu}(\gamma_{\mu}\gamma_{\nu}+2\delta_{\nu\mu})f_{\mu}+M\psi=0\Rightarrow\lambda_1\hat{\partial}\psi-\frac{\lambda_4}{2i}(\partial_{\mu}-\frac{1}{4}\hat{\partial}\gamma_{\mu})f_{\mu}+M\psi=0, \tag{24}$$

where ψ is a bispinor, and f_{μ} is a vector-bispinor. Similarly we derive the following two equations

$$\lambda_6\hat{\partial}\varphi-\frac{\lambda_8}{2i}(\partial_{\mu}-\frac{1}{4}\hat{\partial}\gamma_{\mu})f_{\mu}+M\varphi=0, \tag{25}$$

$$\lambda_{11}\hat{\partial}\Phi-\frac{\lambda_{12}}{2i}(\partial_{\mu}-\frac{1}{4}\hat{\partial}\gamma_{\mu})f_{\mu}+M\Phi=0. \tag{26}$$

Now consider equation

$$\frac{1}{2}[\partial^{\dot{a}}_{c}(\lambda_{13}\psi^{\dot{b}}+\lambda_{14}\varphi^{\dot{b}}+\lambda_{15}\Phi^{\dot{b}})+\partial^{\dot{b}}_{c}(\lambda_{13}\psi^{\dot{a}}+\lambda_{14}\varphi^{\dot{a}}+\lambda_{15}\Phi^{\dot{a}})]+Mf_{c}^{(\dot{a}\dot{b})}=0.$$

Allowing for (21), we obtain

$$\frac{1}{2}[\partial^{\dot{a}}_{c}(\lambda_{13}\psi^{\dot{b}}+\lambda_{14}\varphi^{\dot{b}}+\lambda_{15}\Phi^{\dot{b}})+\partial^{\dot{b}}_{c}(\lambda_{13}\psi^{\dot{a}}+\lambda_{14}\varphi^{\dot{a}}+\lambda_{15}\Phi^{\dot{a}})]+\frac{M}{2}[(\sigma^{\mu})^{\dot{a}}_{c}f^{\dot{b}}_{\mu}+(\sigma^{\mu})^{\dot{b}}_{c}f^{\dot{a}}_{\mu}]=0.$$

Let us multiplying the last equation by $(\sigma^{\lambda})^{c}_{\dot{a}}$

$$\frac{\lambda_{13}}{2}[(\sigma^{\lambda})^{c}_{\dot{a}}\partial^{\dot{a}}_{c}\psi^{\dot{b}}+(\sigma^{\lambda})^{c}_{\dot{a}}\partial^{\dot{b}}_{c}\psi^{\dot{a}}]+\frac{\lambda_{14}}{2}[(\sigma^{\lambda})^{c}_{\dot{a}}\partial^{\dot{a}}_{c}\varphi^{\dot{b}}+(\sigma^{\lambda})^{c}_{\dot{a}}\partial^{\dot{b}}_{c}\varphi^{a}]+$$

$$+\frac{\lambda_{15}}{2}[(\sigma^{\lambda})^{c}_{\dot{a}}\partial^{\dot{a}}_{c}\Phi^{\dot{b}}+(\sigma^{\lambda})^{c}_{\dot{a}}\partial^{\dot{b}}_{c}\Phi^{a}]+\frac{M}{2}[(\sigma^{\lambda})^{c}_{\dot{a}}(\sigma^{\mu})^{\dot{a}}_{c}f^{\dot{b}}_{\mu}+(\sigma^{\lambda})^{c}_{\dot{a}}(\sigma^{\mu})^{\dot{b}}_{c}f^{\dot{a}}_{\mu}]=0,$$

or

$$\frac{\lambda_{13}}{2}[-(\sigma^{\lambda})^{\dot{a}c}\partial_{c\dot{a}}\psi^{\dot{b}}-\partial^{\dot{b}c}(\sigma^{\lambda})_{c\dot{a}}\psi^{\dot{a}}]+\frac{\lambda_{14}}{2}[-(\sigma^{\lambda})^{\dot{a}c}\partial_{c\dot{a}}\varphi^{\dot{b}}-\partial^{\dot{b}c}(\sigma^{\lambda})_{c\dot{a}}\varphi^{\dot{a}}]+$$

$$+\frac{\lambda_{15}}{2}[-(\sigma^{\lambda})^{\dot{a}c}\partial_{c\dot{a}}\Phi^{\dot{b}}-\partial^{\dot{b}c}(\sigma^{\lambda})_{c\dot{a}}\Phi^{\dot{a}}]+\frac{M}{2}[-(\sigma^{\lambda})^{\dot{a}c}(\sigma^{\mu})_{c\dot{a}}f^{\dot{b}}_{\mu}-(\sigma^{\mu})^{\dot{b}c}(\sigma^{\lambda})_{c\dot{a}}f^{\dot{a}}_{\mu}]=0.$$

Whence taking in mind relations (22), (23), we get

$$\lambda_{13}[\frac{2}{i}\delta_{\lambda\nu}\partial_{\nu}\psi^{\dot{b}}-\partial^{\dot{b}c}(\sigma^{\lambda})_{c\dot{a}}\psi^{\dot{a}}]+\lambda_{14}[\frac{2}{i}\delta_{\lambda\nu}\partial_{\nu}\varphi^{\dot{b}}-\partial^{\dot{b}c}(\sigma^{\lambda})_{c\dot{a}}\varphi^{\dot{a}}]+$$

$$+\lambda_{15}[\frac{2}{i}\delta_{\lambda\nu}\partial_{\nu}\Phi^{\dot{b}}-\partial^{\dot{b}c}(\sigma^{\lambda})_{c\dot{a}}\Phi^{\dot{a}}]+M[2\delta_{\alpha\mu}f^{\dot{b}}_{\mu}-(\sigma^{\mu})^{\dot{b}c}(\sigma^{\lambda})_{c\dot{a}}f^{\dot{a}}_{\mu}]=0,$$

or

$$\lambda_{13}\frac{1}{i}\partial_{\nu}[2\delta_{\lambda\nu}\psi^{\dot{b}}-(\sigma^{\nu})^{\dot{b}c}(\sigma^{\lambda})_{c\dot{a}}\psi^{\dot{a}}]+\lambda_{14}\frac{1}{i}\partial_{\nu}[2\delta_{\lambda\nu}\varphi^{\dot{b}}-(\sigma^{\nu})^{\dot{b}c}(\sigma^{\lambda})_{c\dot{a}}\varphi^{\dot{a}}]+$$

$$+\lambda_{15}\frac{1}{i}\partial_{\nu}[2\delta_{\lambda\nu}\Phi^{\dot{b}}-(\sigma^{\nu})^{\dot{b}c}(\sigma^{\lambda})_{c\dot{a}}\Phi^{\dot{a}}]+M[2\delta_{\lambda\mu}f^{\dot{b}}_{\mu}-(\sigma^{\mu})^{\dot{b}c}(\sigma^{\lambda})_{c\dot{a}}f^{\dot{a}}_{\mu}]=0. \tag{27}$$

Similarly, the equation

$$\frac{1}{2}[\partial_a^{\dot c}(\lambda_{13}\psi_b+\lambda_{14}\varphi_b+\lambda_{15}\Phi_b)+\partial_b^{\dot c}(\lambda_{13}\psi_a+\lambda_{14}\varphi_a+\lambda_{15}\Phi_a)]+Mf_{(ab)}^{\dot c}=0$$

is reduced to the form

$$\lambda_{13}\frac{1}{i}\partial_\nu[2\delta_{\lambda\nu}\psi_b-(\sigma^\nu)_{b\dot c}(\sigma^\lambda)^{\dot c a}\psi_a]+\lambda_{14}\frac{1}{i}\partial_\nu[2\delta_{\lambda\nu}\varphi_b-(\sigma^\nu)_{b\dot c}(\sigma^\lambda)^{\dot c a}\varphi_a]+$$

$$+\lambda_{15}\frac{1}{i}\partial_\nu[2\delta_{\lambda\nu}\Phi_b-(\sigma^\nu)_{\dot b c}(\sigma^\lambda)^{c\dot a}\Phi_a]+M[2\delta_{\lambda\nu}f_{b\mu}-(\sigma^\mu)_{b\dot c}(\sigma^\lambda)^{\dot c a}f_{a\mu}]=0. \quad (28)$$

Equations (27), (28) may be joined into the one

$$\frac{1}{i}\partial_\nu\begin{vmatrix}2\delta_{\lambda\nu}\delta_{\dot a}^{\dot b}-(\sigma^\nu)^{\dot b c}(\sigma^\lambda)_{c\dot a} & 0\\ 0 & 2\delta_{\lambda\nu}\delta_b^a-(\sigma^\nu)_{b\dot c}(\sigma^\lambda)^{\dot c a}\end{vmatrix}[\lambda_{13}\begin{vmatrix}\psi^{\dot a}\\ \psi_a\end{vmatrix}+\lambda_{14}\begin{vmatrix}\varphi^{\dot a}\\ \varphi_a\end{vmatrix}+\lambda_{15}\begin{vmatrix}\Phi^{\dot a}\\ \Phi_a\end{vmatrix}]+$$

$$+M\begin{vmatrix}2\delta_{\lambda\mu}\delta_{\dot a}^{\dot b}-(\sigma^\mu)^{\dot b c}(\sigma^\lambda)_{c\dot a} & 0\\ 0 & 2\delta_{\lambda\mu}\delta_b^a-(\sigma^\mu)_{b\dot c}(\sigma^\lambda)^{\dot c a}\end{vmatrix}\begin{vmatrix}f_\mu^{\dot a}\\ \Phi_{a\mu}\end{vmatrix}=0,$$

or differently

$$\frac{1}{i}\partial_\nu(2\delta_{\lambda\nu}+\gamma_\nu\gamma_\lambda)[\lambda_{13}\psi+\lambda_{14}\varphi+\lambda_{15}\Phi]+M(2\delta_{\lambda\mu}+\gamma_\mu\gamma_\lambda)f_\mu=0.$$

Thus we arrive at the equation

$$M(2\delta_{\lambda\mu}-\frac{1}{4}\gamma_\lambda\gamma_\mu)f_\mu-i(\partial_\lambda-\frac{1}{4}\gamma_\lambda\hat\partial)[\lambda_{13}\psi+\lambda_{14}\varphi+\lambda_{15}\Phi]=0. \quad (29)$$

So, the system of spin-tensor equations has the form (24)−(26), (29); where we should take into account the constraints on parameters λ_i (16)–(20).

5. Minimal system of equations, the free particle case

From eq. (29) let us express the quantity $(\delta_{\lambda\mu}-\frac{1}{4}\gamma_\lambda\gamma_\mu)f_\mu$:

$$(\delta_{\lambda\mu}-\frac{1}{4}\gamma_\lambda\gamma_\mu)f_\mu=\frac{i}{M}(\partial_\lambda-\frac{1}{4}\gamma_\lambda\hat\partial)[\lambda_{13}\psi+\lambda_{14}\varphi+\lambda_{15}\Phi];$$

whence we derive

$$(\partial_\mu-\frac{1}{4}\hat\partial\gamma_\mu)f_\mu=\frac{i}{M}\cdot\frac{3}{4}\Box(\lambda_{13}\psi+\lambda_{14}\varphi+\lambda_{15}\Phi),\quad \partial_\mu\partial_\mu=\Box. \quad (30)$$

Taking into account relations (30) in equations (24)–(26), we obtain

$$(M+\lambda_1\hat\partial\Psi)+\frac{3}{2M}\lambda_4\Box(\lambda_{13}\psi+\lambda_{14}\varphi+\lambda_{15}\Phi)=0, \quad (31)$$

$$(M+\lambda_6\hat\partial)\varphi+\frac{3}{2M}\lambda_8\Box(\lambda_{13}\psi+\lambda_{14}\varphi+\lambda_{15}\Phi)=0, \quad (32)$$

$$(M+\lambda_{11}\hat\partial)\Phi+\frac{3}{2M}\lambda_{12}\Box(\lambda_{13}\psi+\lambda_{14}\varphi+\lambda_{15}\Phi)=0. \quad (33)$$

Let us act on equations (31) – (33) correspondingly by the operators,

$$\lambda_{13}(M+\lambda_6\hat{\partial})(M+\lambda_{11}\hat{\partial}), \quad \lambda_{14}(M+\lambda_1\hat{\partial})(M+\lambda_{11}\hat{\partial}), \quad \lambda_{15}(M+\lambda_1\hat{\partial})(M+\lambda_6\hat{\partial});$$

and then sum the results. In this way, we arrive at the equation

$$\{M^3+M^2\hat{\partial}+M(\lambda_1\lambda_6+\lambda_1\lambda_{11}+\lambda_6\lambda_{11})\Box+\lambda_1\lambda_6\lambda_{11}\Box\hat{\partial}\}(\lambda_{13}\Psi+\lambda_{14}\varphi+\lambda_{15}\Phi)+$$

$$+\frac{3}{2M}\{(\lambda_4\lambda_{13}+\lambda_8\lambda_{14}+\lambda_{12}\lambda_{15})M^2+M[\lambda_4\lambda_{13}(\lambda_6+\lambda_{11})+\lambda_8\lambda_{14}(\lambda_1+\lambda_{11})+\lambda_{12}\lambda_{15}(\lambda_1+\lambda_6)]\hat{\partial}+$$

$$+(\lambda_4\lambda_6\lambda_{11}\lambda_{13}+\lambda_1\lambda_8\lambda_{11}\lambda_{14}+\lambda_1\lambda_6\lambda_{12}\lambda_{15})\Box\}\Box(\lambda_{13}\psi+\lambda_{14}\varphi+\lambda_{15}\Phi)=0.$$

Allowing for the restrictions (17)–(20), we derive the equation

$$M^2\{\hat{\partial}+M\}(\lambda_{13}\psi+\lambda_{14}\varphi+\lambda_{15}\Phi)=0,$$

which coincides with the Dirac equation for a free particle

$$(\hat{\partial}+M)\Psi=0, \quad \Psi=\lambda_{13}\psi+\lambda_{14}\varphi+\lambda_{15}\Phi.$$

6. The minimal system of equations in presence of electromagnetic fields

Let us start with the first order system in presence of the electromagnetic fields

$$(M+\lambda_1\hat{D})\psi-i2\lambda_4(D_\mu-\frac{1}{4}\hat{D}\gamma_\mu)f_\mu=0, \quad (34)$$

$$(M+\lambda_6\hat{D})\varphi-i2\lambda_8(D_\mu-\frac{1}{4}\hat{D}\gamma_\mu)f_\mu=0, \quad (35)$$

$$(M+\lambda_{11}\hat{D})\Phi-i2\lambda_{12}(D_\mu-\frac{1}{4}\hat{D}\gamma_\mu)f_\mu=0, \quad (36)$$

$$M(\delta_{\lambda\mu}-\frac{1}{4}\gamma_\lambda\gamma_\mu)f_\mu-i(D_\lambda-\frac{1}{4}\gamma_\lambda\hat{D})[\lambda_{13}\psi+\lambda_{14}\varphi+\lambda_{15}\Phi]=0; \quad (37)$$

where $D_\mu=\partial_\mu-ieA_\mu$.

From the last equation in the system (34) – (37), let us express the quantity $(\delta_{\lambda\mu}-\frac{1}{4}\gamma_\lambda\gamma_\mu)f_\mu$:

$$(\delta_{\lambda\mu}-\frac{1}{4}\gamma_\lambda\gamma_\mu)f_\mu=\frac{i}{M}(D_\lambda-\frac{1}{4}\gamma_\lambda\hat{D})(\lambda_{13}\psi+\lambda_{14}\varphi+\lambda_{15}\Phi).$$

Act on this equation by operator D_λ:

$$(D_\mu-\frac{1}{4}\hat{D}\gamma_\mu)f_\mu=\frac{i}{M}(D^2-\frac{1}{4}\hat{D}\hat{D})(\lambda_{13}\Psi+\lambda_{14}\varphi+\lambda_{15}\Phi), \quad (38)$$

where $D^2=D_\mu D_\mu$. Allowing for expression (38), we can rewrite equations (34) – (36) differently

$$(M+\lambda_1\hat{D})\psi+\frac{2}{M}\lambda_4(D^2-\frac{1}{4}\hat{D}\hat{D})(\lambda_{13}\psi+\lambda_{14}\varphi+\lambda_{15}\Phi)=0, \quad (39)$$

$$(M+\lambda_6\hat{D})\varphi+\frac{2}{M}\lambda_8(D^2-\frac{1}{4}\hat{D}\hat{D})(\lambda_{13}\psi+\lambda_{14}\varphi+\lambda_{15}\Phi)=0, \quad (40)$$

$$(M+\lambda_{11}\hat{D})\Phi+\frac{2}{M}\lambda_{12}(D^2-\frac{1}{4}\hat{D}\hat{D})(\lambda_{13}\psi+\lambda_{14}\varphi+\lambda_{15}\Phi)=0. \quad (41)$$

Act on equations (39)–(41) correspondingly by the following operators

$$\lambda_{13}(M+\lambda_6\hat{D})(M+\lambda_{11}\hat{D}), \quad \lambda_{14}(M+\lambda_1\hat{D})(M+\lambda_{11}\hat{D}), \quad \lambda_{15}(M+\lambda_1\hat{D})(M+\lambda_6\hat{D});$$

this results in

$$\lambda_{13}\{M^3+M^2\hat{D}+M(\lambda_1\lambda_6+\lambda_1\lambda_{11}+\lambda_6\lambda_{11})\hat{D}\hat{D}+$$
$$+\lambda_1\lambda_6\lambda_{11}\hat{D}\hat{D}\hat{D}\}\psi+\frac{2}{M}\lambda_4\lambda_{13}\{M^2+M(\lambda_6+\lambda_{11})\hat{D}+$$
$$+\lambda_6\lambda_{11}\hat{D}\hat{D}\}(D^2-\frac{1}{4}\hat{D}\hat{D})(\lambda_{13}\psi+\lambda_{14}\varphi+\lambda_{15}\Phi)=0,$$
$$\lambda_{14}\{M^3+M^2\hat{D}+M(\lambda_1\lambda_6+\lambda_1\lambda_{11}+\lambda_6\lambda_{11})\hat{D}\hat{D}+$$
$$+\lambda_1\lambda_6\lambda_{11}\hat{D}\hat{D}\hat{D}\}\varphi+\frac{2}{M}\lambda_8\lambda_{14}\{M^2+M(\lambda_1+\lambda_{11})\hat{D}+$$
$$+\lambda_1\lambda_{11}\hat{D}\hat{D}\}(D^2-\frac{1}{4}\hat{D}\hat{D})(\lambda_{13}\psi+\lambda_{14}\varphi+\lambda_{15}\Phi)=0,$$
$$\lambda_{15}\{M^3+M^2\hat{D}+M(\lambda_1\lambda_6+\lambda_1\lambda_{11}+\lambda_6\lambda_{11})\hat{D}\hat{D}+$$
$$+\lambda_1\lambda_6\lambda_{11}\hat{D}\hat{D}\hat{D}\}\Phi+\frac{2}{M}\lambda_{12}\lambda_{15}\{M^2+M(\lambda_1+\lambda_6)\hat{D}+$$
$$+\lambda_1\lambda_6\hat{D}\hat{D}\}(D^2-\frac{1}{4}\hat{D}\hat{D})(\lambda_{13}\psi+\lambda_{14}\varphi+\lambda_{15}\Phi)=0.$$

Let us sum this three equations

$$\{M^3+M^2\hat{D}+M(\lambda_1\lambda_6+\lambda_1\lambda_{11}+\lambda_6\lambda_{11})\hat{D}\hat{D}+\lambda_1\lambda_6\lambda_{11}\hat{D}\hat{D}\hat{D}\}(\lambda_{13}\psi+\lambda_{14}\varphi+\lambda_{15}\Phi)+$$
$$+\frac{2}{M}\{M^2(\lambda_4\lambda_{13}+\lambda_8\lambda_{14}+\lambda_{12}\lambda_{15})+M[\lambda_4\lambda_{13}(\lambda_6+\lambda_{11})+$$
$$+\lambda_8\lambda_{14}(\lambda_1+\lambda_{11})+\lambda_{12}\lambda_{15}(\lambda_1+\lambda_6)]\hat{D}+[\lambda_4\lambda_6\lambda_{11}\lambda_{13}+\lambda_1\lambda_8\lambda_{11}\lambda_{14}+$$
$$+\lambda_1\lambda_6\lambda_{12}\lambda_{15}]\hat{D}\hat{D}\}(D^2-\frac{1}{4}\hat{D}\hat{D})(\lambda_{13}\psi+\lambda_{14}\varphi+\lambda_{15}\Phi)=0. \quad (42)$$

The last term in eq. (42) vanishes identically due to restriction (20). Thus, we have

$$\{M^3+M^2\hat{D}+M(\lambda_1\lambda_6+\lambda_1\lambda_{11}+\lambda_6\lambda_{11})\hat{D}\hat{D}+\lambda_1\lambda_6\lambda_{11}\hat{D}\hat{D}\hat{D}+$$
$$+2M(\lambda_4\lambda_{13}+\lambda_8\lambda_{14}+\lambda_{12}\lambda_{15})(D^2-\frac{1}{4}\hat{D}\hat{D})+$$
$$+2[\lambda_4\lambda_{13}(\lambda_6+\lambda_{11})+\lambda_8\lambda_{14}(\lambda_1+\lambda_{11})+\lambda_{12}\lambda_{15}(\lambda_1+\lambda_6)]\times$$
$$\times\hat{D}(D^2-\frac{1}{4}\hat{D}\hat{D})\}(\lambda_{13}\psi+\lambda_{14}\varphi+\lambda_{15}\Phi)=0.$$

Taking in mind the identity

$$\hat{D}\hat{D}=D^2-ieF_{[\mu\nu]}I_{[\mu\nu]}, \quad where \quad F_{[\mu\nu]}=\partial_\mu A_\nu-\partial_\nu A_\mu, \quad j_{[\mu\nu]}=\frac{1}{4}(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu),$$

we transform the previous equation to the following one

$$\{M^3+M^2\hat{D}+M(\lambda_1\lambda_6+\lambda_1\lambda_{11}+\lambda_6\lambda_{11})D^2+\frac{3}{2}M(\lambda_4\lambda_{13}+$$

$$+\lambda_8\lambda_{14}+\lambda_{12}\lambda_{15})D^2-ieM(\lambda_1\lambda_6+\lambda_1\lambda_{11}+\lambda_6\lambda_{11})F_{[\mu\nu]}j_{[\mu\nu]}+$$

$$+\frac{ie}{2}M(\lambda_4\lambda_{13}+\lambda_8\lambda_{14}+\lambda_{12}\lambda_{15})F_{[\mu\nu]}j_{[\mu\nu]}+$$

$$+[\lambda_1\lambda_6\lambda_{11}+\frac{3}{2}(\lambda_4\lambda_{13}(\lambda_6+\lambda_{11})+\lambda_8\lambda_{14}(\lambda_1+\lambda_{11})+\lambda_{12}\lambda_{15}(\lambda_1+\lambda_6))]\times$$

$$\times\hat{D}D^2+ie[-\lambda_1\lambda_6\lambda_{11}+\frac{1}{2}(\lambda_4\lambda_{13}(\lambda_6+\lambda_{11})+\lambda_8\lambda_{14}(\lambda_1+\lambda_{11})+$$

$$+\lambda_{12}\lambda_{15}(\lambda_1+\lambda_6)]\hat{D}F_{[\mu\nu]}j_{[\mu\nu]}\}(\lambda_{13}\psi+\lambda_{14}\varphi+\lambda_{15}\Phi)=0,$$

or

$$\{M^3+M^2\hat{D}+2ieM(\lambda_4\lambda_{13}+\lambda_8\lambda_{14}+\lambda_{12}\lambda_{15})F_{[\mu\nu]}j_{[\mu\nu]}+$$

$$+[(\lambda_1\lambda_6+\lambda_1\lambda_{11}+\lambda_6\lambda_{11})+\frac{3}{2}(\lambda_1\lambda_4\lambda_{13}+\lambda_6\lambda_8\lambda_{14}+\lambda_{11}\lambda_{12}\lambda_{15})+$$

$$+\frac{3}{2}\lambda_4\lambda_{13}(\lambda_6+\lambda_{11})+\frac{3}{2}\lambda_8\lambda_{14}(\lambda_1+\lambda_{11})+\frac{3}{2}\lambda_{12}\lambda_{15}(\lambda_1+\lambda_{11})]\hat{D}D^2+$$

$$+ie[-(\lambda_1\lambda_6+\lambda_1\lambda_{11}+\lambda_6\lambda_{11})-\frac{3}{2}(\lambda_1\lambda_4\lambda_{13}+\lambda_6\lambda_8\lambda_{14}+\lambda_{11}\lambda_{12}\lambda_{15})+\frac{1}{2}\lambda_4\lambda_{13}(\lambda_6+\lambda_{11})+$$

$$+\frac{1}{2}\lambda_8\lambda_{14}(\lambda_1+\lambda_{11})+\frac{1}{2}\lambda_{12}\lambda_{15}(\lambda_1+\lambda_6)]\hat{D}F_{[\mu\nu]}j_{[\mu\nu]}\}\cdot(\lambda_{13}\psi+\lambda_{14}\varphi+\lambda_{15}\Phi)=0.$$

Thus, we arrive at the equation

$$\{M^3+M^2\hat{D}+2ieM(\lambda_4\lambda_{13}+\lambda_8\lambda_{14}+\lambda_{12}\lambda_{15})F_{[\mu\nu]}j_{[\mu\nu]}+$$

$$+2ie[(\lambda_6+\lambda_{11})\lambda_4\lambda_{13}+(\lambda_1+\lambda_{11})\lambda_8\lambda_{14}+(\lambda_1+\lambda_6)\lambda_{12}\lambda_{15}]\hat{D}F_{[\mu\nu]}j_{[\mu\nu]}\}\times$$

$$\times(\lambda_{13}\Psi+\lambda_{14}\varphi+\lambda_{15}\Phi)=0.$$

Therefore, in presence of the external magnetic fields, we have the generalized Dirac-like equation

$$\{\hat{D}+M+2\frac{ie}{M}(\lambda_4\lambda_{13}+\lambda_8\lambda_{14}+\lambda_{12}\lambda_{15})F_{[\mu\nu]}j_{[\mu\nu]}+$$

$$+2\frac{ie}{M^2}[(\lambda_6+\lambda_{11})\lambda_4\lambda_{13}+(\lambda_1+\lambda_{11})\lambda_8\lambda_{14}+(\lambda_1+\lambda_6)\lambda_{12}\lambda_{15}]\hat{D}F_{[\mu\nu]}j_{[\mu\nu]}\}\Psi=0 \quad (43)$$

with respect to the bispinor function

$$\Psi=\lambda_{13}\Psi+\lambda_{14}\varphi+\lambda_{15}\Phi;$$

equation (43) contains two addition interaction terms.

The first term

$$2\frac{ie}{M}(\lambda_4\lambda_{13}+\lambda_8\lambda_{14}+\lambda_{12}\lambda_{15})F_{[\mu\nu]}j_{[\mu\nu]}=ie\mu F_{[\mu\nu]}j_{[\mu\nu]}$$

relates to the anomalous magnetic moment of the particle; the second term

$$2\frac{ie}{M^2}[(\lambda_6+\lambda_{11})\lambda_4\lambda_{13}+(\lambda_1+\lambda_{11})\lambda_8\lambda_{14}+(\lambda_1+\lambda_6)\lambda_{12}\lambda_{15}]\hat{D}F_{[\mu\nu]}j_{[\mu\nu]}=ie\sigma\hat{D}F_{[\mu\nu]}j_{[\mu\nu]}$$

may be considered as referring to some additional characteristics of the particle (polarizability).

Conclusions

We have constructed a generalized relativistic equation for a spin 1/2 particle with two additional characteristics in presence of external electromagnetic fields. After eliminating the accessory variables of the complete wave function, we have derived the generalized Dirac-like equation, the last includes two additional interaction terms which are interpreted as related to the anomalous magnetic moment and the polarizability of the particle. This equation may be a base for experimental testing the intrinsic structure of the spin 1/2 particle.

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