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Simple Instanton Solutions in Quantum Mechanics and Quantum Field Theory

The simplest instanton solutions appearing in one-dimensional quantum mechanical tasks with rectangular and linear potentials are considered. Using the method of Feynman functional integrals in imaginary time, the well-known expressions for the probabilities of sub-barrier tunnelling are reproduced with pre-exponential accuracy. An instanton solution is obtained for two-dimensional scalar field theory (in a limited spatial region) with a classically degenerated vacuum states; the probability of a tunnel transition is calculated in WKB approximation. A soliton solution is obtained for three-dimensional scalar field theory in space with cylinder geometry.

Key words: *quantum tunneling, potential barrier, cold electron emission, scalar field theory with double-well potential, instanton, soliton.*

ПРОСТЕЙШИЕ ИНСТАНТОННЫЕ РЕШЕНИЯ В КВАНТОВОЙ МЕХАНИКЕ И КВАНТОВОЙ ТЕОРИИ ПОЛЯ

Рассмотрены простейшие инстантонные решения, возникающие в одномерных квантово-механических задачах с прямоугольным и линейным потенциалами. Методом фейнмановских функциональных интегралов во мнимом времени воспроизведены известные выражения для вероятностей подбарьерного туннелирования с предэкспоненциальной точностью. Получено инстантонное решение для двумерной скалярной полевой теории (в ограниченной пространственной области) с классически вырожденными вакуумными состояниями; вычислена вероятность туннельного перехода в ВКБ-приближении. Получено солитонное решение для трехмерной скалярной полевой теории в пространстве с геометрией цилиндра.

Ключевые слова: *квантовое туннелирование, потенциальный барьер, холодная электронная эмиссия, скалярная полевая теория с двуямным потенциалом, инстантон, солитон.*

Introduction

Instantons [1] are non-trivial solutions of ***classical*** Lagrange-Euler equations with a finite action in the Euclidean space, or, what is the same, in imaginary time [2]. Instantons acquire physical meaning in the ***quantum*** version of the theory. Here, using the Feynman formalism of path integrals [3], instantons describe quantum tunnelling between classically degenerated vacuum states [4]. The term «instanton» (from the English «instant»/«instant») was proposed by G. t'Hooft [5; 6] (in the original work of A. M. Polyakov et al. [1], such solutions were called «pseudoparticles»), since in physically interesting theories (such as quantum chromodynamics or Glashow – Weinberg – Salam theory) such tunnelling, if it occurs, occurs very rapidly («instantly»), since the potential barrier turns out to be large and, according to the Heisenberg uncertainty relation ΔEΔt~ħ, the tunnelling time Δt turns out to be very small. This leads to an extremely low probability of instanton processes: for strong interactions this quantity is of the order of $\exp(-4\pi/\alpha_S) \sim 10^{-7}$, where $\alpha_S(0,6\,GeV) \approx 0,8$ at $\Lambda_{QCD} = 200\,MeV$. The scale of the running constant of strong interactions corresponds to the characteristic size of QCD instantons (~1/3 Fm) obtained from phenomenological considerations (see, for example, [7]) and analysis of the Shifman – Weinstein – Zakharov sum rule [8]. For weak interactions, the situation is even more pessimistic: the corresponding probability is proportional to the suppressing exponent $\exp(-4\pi/\alpha_W) \sim 10^{-170}$, where $\alpha_W = g^2/4\pi = \alpha/\sin^2\theta_W \approx 0,032$. Such smallness can be compensated in processes at high energies or high temperatures, for example, in the processes of collision of hadrons at LHC or heavy ions with the formation of a quark-gluon plasma at RHIC facility [9; 10]. Experimental confirmation of the existence of instanton processes will serve as an important confirmation of the correctness of the Standard Model and our knowledge of the world of elementary particles and fundamental interactions, along with the experimental discovery of massive vector bosons, top quark and Higgs boson. It is important to note that the existence of instanton effects makes it possible to explain at the phenomenological level the problem of the baryon asymmetry of the visible part of the Universe [11] and the problem of spontaneous violation of the chiral invariance of trong interactions [12].

Traditionally, an introduction to instanton physics begins with a consideration of one-dimensional ***quantum-mechanical*** tasks with double-well or periodic potentials, as well as task of the motion of a particle on a circle [4]. At the same time, more obvious and traditional for student courses in quantum mechanics, tasks of passing a rectangular potential barrier and task of cold emission of electrons from a metal are ignored. In this paper, this methodological gap is filled (Sections 1, 2). An important application of the instanton method is the study of the chaotic behaviour of quantum mechanical systems with enforced tunnelling transitions [13; 14].

In ***field theory***, consideration of instanton solutions traditionally begins with rather complex models: the Abelian Higgs model in space-time (1 + 1) or SU(2) Yang – Mills gauge theory in four dimensions. The simplest field theory model could be a scalar field theory in space-time (1 + 1) with a degenerate vacuum state (for example, a sine-Gordon model or a theory with a double-well potential). However, the existence of instantons describing tunnelling transitions between classically degenerate vacuums is forbidden [15] (the Derrick – Hobart theorem), see also [4]. At the same time, there are at least two cases in which the Derrick – Hobart «no-go» theorem is not valid. First, this is the case of a scalar field theory in a ***limited spatial volume*** [16]. In the case of two dimensions, instanton solutions are known analytically [17]. Secondly, the quantum-field consideration of tunnelling effects does not necessarily imply the presence of classical solutions in imaginary time (instantons). That is, there can be transitions that are not described by instantons, associated with tunnelling from one vacuum state to another and back in a finite region of space in a finite time. Such processes require finite energy and can go according to the Heisenberg uncertainty relation. Sections 3 of this article are devoted to consideration of tunnelling effects in scalar field models.

It should be noted that classical solutions in scalar field models are considered to describe the processes of false vacuum decay. These are «bounce» solutions [16].

Also, soliton solution is obtained for three-dimensional scalar field theory in space with cylinder geometry (Sections 4).

1. Instantons in the task of passing a rectangular potential barrier

Let's consider one of the simplest quantum mechanical problems, one-dimensional task of passing a rectangular potential barrier (Figure 1). The Lagrangian has the form:

$$L = \frac{m}{2}\left(\frac{dx}{dt}\right)^2 - V(x), \qquad V(x) = \begin{cases} 0, & x < 0 \\ V_0, & 0 < x < a \\ 0, & x > 0 \end{cases}. \tag{1}$$

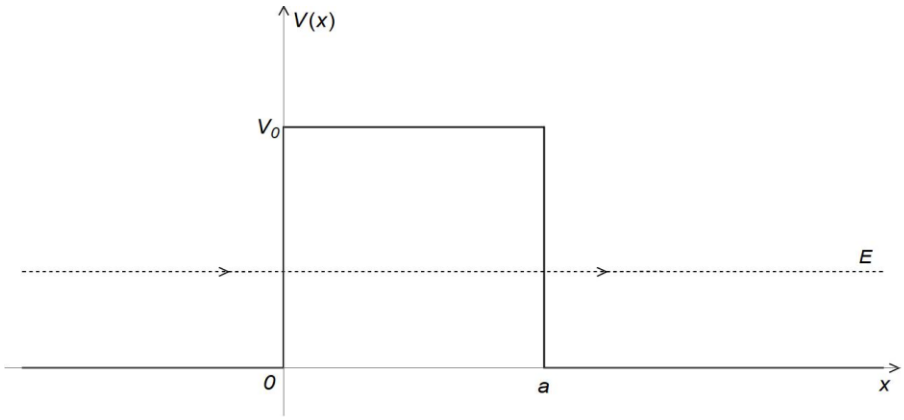

Figure 1. – Rectangular potential barrier

Here m is the mass of the particle, V_0 is the height of the potential barrier, and a is its width. This standard task can be solved exactly on the basis of stationary Schrödinger equation:

$$\hat{H}\varphi(x) = E\varphi(x), \quad \hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x), \quad E < V_0. \tag{2}$$

The transition coefficient has the form [18]:

$$D(E) = \frac{4E(V_0 - E)}{4E(V_0 - E) + V_0^2 \operatorname{sh}^2\left(\frac{a}{\hbar}\sqrt{2m(V_0 - E)}\right)}. \tag{3}$$

If $E << V_0$ and $a >> \hbar$ the expression (3) is simplified:

$$D(E) \approx 16\frac{E}{V_0}\exp\left(-\frac{2a}{\hbar}\sqrt{2m(V_0 - E)}\right). \tag{4}$$

Often, the pre-exponential factor is neglected for estimation, considering it to be approximately equal to one. This estimate is obviously a rough one:

$$D(E) \propto \exp\left(-\frac{2a}{\hbar}\sqrt{2m(V_0 - E)}\right). \tag{5}$$

In this section the solution of this task ***by instanton method*** with pre-exponential accuracy (4) is presented. Let's go to imaginary time (or to Euclidean space):

$$t \to -i\tau, \quad \tau^* = \tau. \tag{6}$$

Where τ is «a time» in Euclidean space. The action corresponding to Lagrangian (1) changes as follows:

$$S=\int_0^{\Delta t}dtL=\int_0^{\Delta t}dt\left(\frac{m}{2}\left(\frac{dx}{dt}\right)^2-V(x)\right)\to i\int_0^T\left(\frac{m}{2}\left(\frac{dx}{d\tau}\right)^2+V(x)\right)d\tau,\quad S_E=\int_0^T\left(\frac{m}{2}\left(\frac{dx}{d\tau}\right)^2+V(x)\right)d\tau. \tag{7}$$

Here S_E is Euclidean action, T – «time» interval in Euclidean space: $\Delta t\to -iT$. Of course, the transition to Euclidean space is only a mathematical technique. Imaginary time has no physical sense. Formally, this corresponds to the classical problem of motion in a rectangular potential well (potential «$-V(x)$», Figure 2).

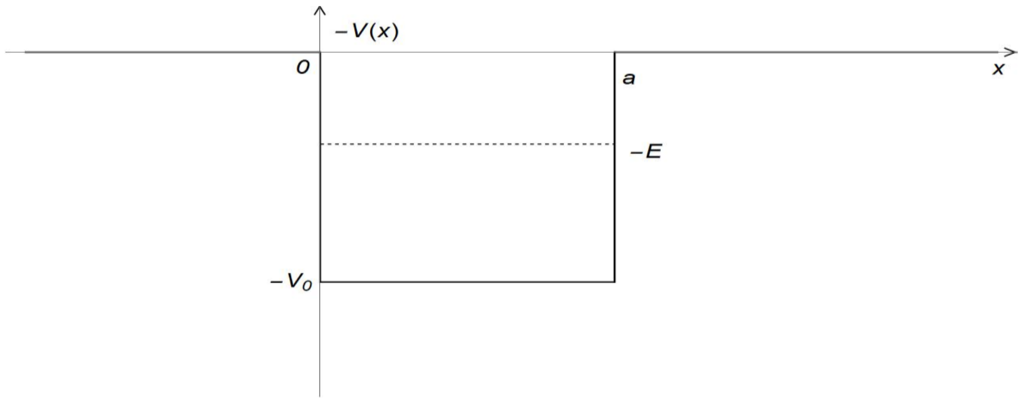

Figure 2. – The problem of a rectangular potential barrier in Euclidean space (formally, the potential «turns over» and turns into a rectangular potential well)

Thus, instanton is a non-trivial classical solution of the equations of motion following from action (7). The easiest way to find it is to write down the Euclidean energy:

$$\frac{m}{2}\left(\frac{dx}{d\tau}\right)^2-V_0=-E,\quad V_0>0,\quad E>0. \tag{8}$$

We are talking about movement in a classically allowed area (Figure 2). The equation is easy to integrate:

$$x^{inst}(\tau,E)=\tau\sqrt{\frac{2}{m}(V_0-E)},\quad x^{inst}(T)=a,\quad x^{inst}(0)=0. \tag{9}$$

The first initial condition immediately gives the Euclidean time of particle movement from one potential wall to another:

$$T=\sqrt{\frac{ma^2}{2(V_0-E)}}. \tag{10}$$

A more important quantity is Euclidean action on the instanton solution (7):

$$S_0\equiv\tilde{S}_E[x^{inst}(\tau,E)]=a\sqrt{2m(V_0-E)}. \tag{11}$$

Here the tilde means truncated action for a period:

$$\tilde{S}_E[x^{inst}(\tau,E)]=S_E[x^{inst}(\tau,E)]-ET. \tag{12}$$

Namely truncated Euclidean action gives the correct contribution to the amplitude [16]. Strictly speaking, the instanton corresponds to the transition «vacuum → vacuum», then the tilde is obviously not needed. At nonzero energies, the corresponding solution is sometimes called instanton-like.

According to Feynman formulation of quantum mechanics (with the help of functional integrals over classical trajectories) the Euclidean amplitude of the transition from the initial to the final state is given by the following expression:

$$< x_f \,|\exp\left(-\hat{H}T\right)|\, x_i >= \int D[x(\tau)]\exp\left(-\tilde{S}_E[x(\tau)]\right). \qquad (13)$$

Here and further $\hbar = c = 1$. The initial conditions in the functional integral are assumed, but usually not written. $D[x(\tau)]$ is the measure of integration in the functional space (more details can be found in the monograph by R. Feynman and A. Hibs [3]).

In the zero approximation with exponential accuracy, the expression for the barrier penetration coefficient (5) is reproduced with exponential accuracy:

$$\begin{gathered}\int ND[x(\tau)]\exp\left(-\tilde{S}_E[x(\tau)]\right) \propto \exp\left(-\tilde{S}_E[x^{inst}(\tau)]\right) = \exp\left(-a\sqrt{2m(V_0-E)}\right),\\ D(E) \propto \exp\left(-2\tilde{S}_E[x^{inst}(\tau)]\right) = \exp\left(-2a\sqrt{2m(V_0-E)}\right).\end{gathered} \qquad (14)$$

Note that the reverse transition from the Euclidean space to the Minkowski space–time, as in many other tasks, is not required.

For more accurate calculations, it is necessary to take into account the first quantum correction to the action on the classical solution. For large values of the Euclidean action $S_E >> 1$, one can write:

$$\begin{gathered}\tilde{S}_E[x(\tau)] \approx \tilde{S}_E[x^{inst}(\tau)] + \delta\, S_E[x^{inst}(\tau)] + \frac{1}{2}\delta^{(2)}S_E[x^{inst}(\tau)] =\\ = a\sqrt{2m(V_0-E)} + \frac{1}{2}\int_0^T d\tau\delta\, x(\tau)\left(-m\partial_\tau^2 + \partial_x^2 V[x^{inst}(\tau)]\right)\delta\, x(\tau)\,.\end{gathered} \qquad (15)$$

The first variation of the action is zero on the classical solution. Assume that there is a set of nonzero eigenvalues ε_n and a set eigenfunctions $y_n(\tau)$ of operator $\hat{O} = -m\frac{\partial^2}{\partial\tau^2} + \frac{\partial^2}{\partial x^2}V[x^{inst}(\tau)] = -m\frac{\partial^2}{\partial\tau^2}$ that form a complete basis:

$$\hat{O}y_n(\tau) = -m\frac{\partial^2}{\partial\tau^2}y_n(\tau) = \varepsilon_n y_n(\tau). \qquad (16)$$

We can write the decomposition:

$$\delta x(\tau) = \sum_n c_n y_n(\tau). \qquad (17)$$

Then the amplitude (13) can be written as follows:

$$< x_f \,|\exp\left(-\hat{H}T\right)|\, x_i >= \frac{B}{\sqrt{\det\hat{O}}}\mathrm{e}^{-a\sqrt{2m(V_0-E)}} = N\mathrm{e}^{-a\sqrt{2m(V_0-E)}}. \qquad (18)$$

We assume that the Jacobian of the transition from functional integration over $\delta x(\tau)$ to integration over c_n enters to the normalization factor B. In order to determine the correct pre-exponential factor N, we consider the exactly solvable task of free motion using the Schrödinger equation and the method of functional integrals:

$$< x_f \,|\exp\left(-\hat{H}T\right)|\, x_i >= \int dp < x_f \,|\exp\left(-(\hat{p}^2/2m)T\right)|\, p >< p\,|\, x_i >=$$
$$= \int \frac{dp}{2\pi}\exp\left(-(p^2/2m)T - ipa\right) = \exp\left(-\frac{a^2 m}{2T}\right)\sqrt{\frac{m}{2\pi T}};$$
$$\int\limits_{x_i}^{x_f} D[x(\tau)]\exp\left(-\int\limits_0^T\left(\frac{m}{2}\left(\frac{dx}{d\tau}\right)^2\right)d\tau\right) = \exp\left(-\frac{a^2 m}{2T}\right)\frac{B}{\sqrt{\det\hat{A}}}. \tag{19}$$

If we calibrate the expression for the amplitude of the tunneling through rectangular barrier (18) according to the known result (19), then the factor N at low energies gives the correct pre-exponential factor:

$$N = \sqrt{\frac{m}{2\pi T}}\sqrt{\frac{\det\hat{A}}{\det\hat{O}}} \approx \frac{4}{a}\sqrt{\frac{E}{V_0}}. \tag{20}$$

Taking this into account, the normalized amplitude (18) leads to the correct expression for the barrier penetration coefficient with pre-exponential accuracy (4).

2. Instanton method for the task of cold emission of electrons from a metal

Let's consider one-dimensional quantum-mechanical task about transition of a linear (triangular) potential (Figure 3), which models the effect of cold electron emission of electrons:

$$L = \frac{m}{2}\left(\frac{dx}{dt}\right)^2 - V(x), \qquad V(x) = \begin{cases} 0 & x < 0 \\ V_0\left(1 - \dfrac{x}{a}\right) & x > 0. \end{cases} \tag{21}$$

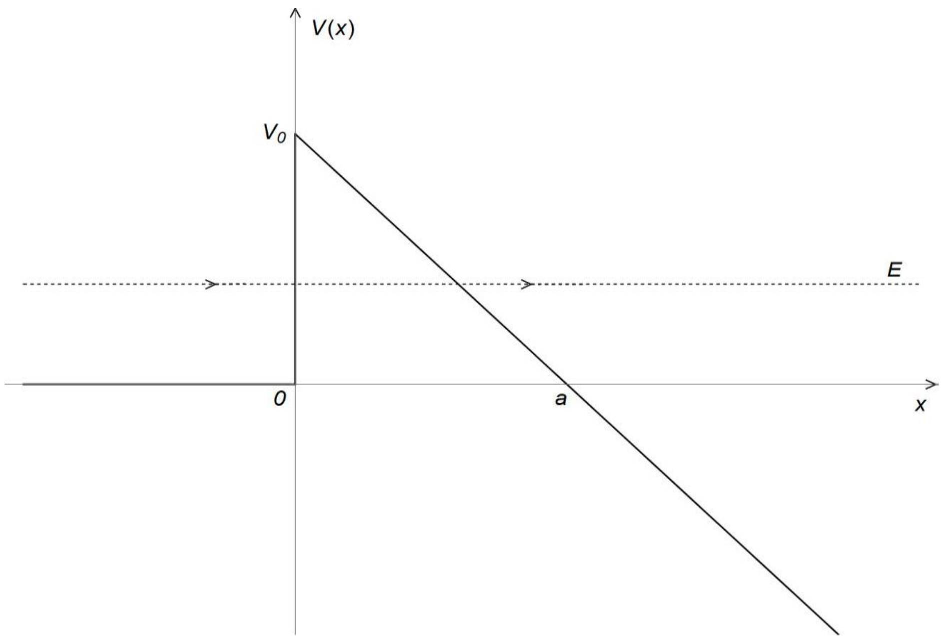

Figure 3. – One-dimensional model of the effect of cold electron emission from metal

Following the reasoning given in Section 1, we obtain the following results.

The Euclidean action of the original task corresponds to an analogous task with an «inverted» potential (Figure 4):

$$S_E = \int_0^T \left(\frac{m}{2}\left(\frac{dx}{d\tau}\right)^2 + V_0\left(1-\frac{x}{a}\right)\right) d\tau, \qquad x(T_0)=a, \quad x(0)=0. \tag{22}$$

The instanton solution that minimizes this action corresponds to the movement of a particle from point 0 to a point a at time T_0 at ***zero energy***:

$$x^{inst}(\tau) = a\left(1-\left(1-\frac{\tau}{T_0}\right)^2\right), \quad T_0 = a\sqrt{\frac{2m}{V_0}}. \tag{23}$$

The Euclidean action on an instanton:

$$S_E[x^{inst}(\tau)] = 2\int_0^{T_0} \frac{m}{2}\left(\frac{dx^{inst}}{d\tau}\right)^2 d\tau = 2\int_0^{T_0} V_0\left(1-\frac{\tau}{T_0}\right)^2 d\tau = \frac{2}{3}a\sqrt{2mV_0}. \tag{24}$$

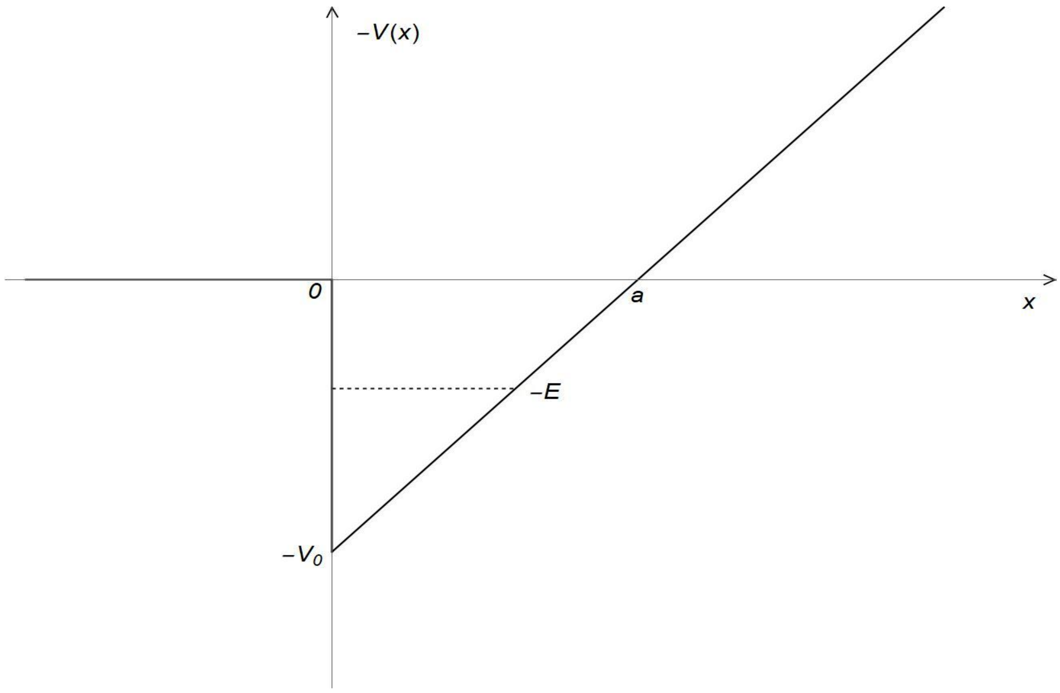

Figure 4. – Euclidean version of the one-dimensional problem of cold electron emission from a metal

Instanton (strictly speaking, instanton-like) solution at ***non-zero*** negative energy – E:

$$x^{inst}(\tau, E) = a\left(1+\frac{E}{V_0}-\left(\sqrt{\frac{V_0-E}{V_0}}-\frac{\tau}{T_0}\right)^2\right). \tag{25}$$

Let us introduce standard notation for the work function $W = V_0 - E$ of electrons from a metal. We take into account that the tangent of the slope of the potential energy graph is related to the force acting from the external field on the elementary charge $-V_0 a^{-1} = -e\mathcal{E}$. In such notation, the instanton solution will take the form:

$$x^{inst}(\tau,W) = a + \frac{E}{e\mathcal{E}} - \left(\sqrt{\frac{W}{e\mathcal{E}}} - \frac{\tau}{T_0}\right)^2, \tag{26}$$

and the truncated action

$$\tilde{S}_E[x^{inst}(\tau,W)] = \frac{2\sqrt{2m}}{3e\mathcal{E}} W^{3/2} \tag{27}$$

determines the exponent in the barrier transmission coefficient

$$D(W) \approx D_0 \exp\left(-\frac{4\sqrt{2m}}{3e\mathcal{E}} W^{3/2}\right). \tag{28}$$

The transmission coefficient determines the main physical value of the effect, the strength of the electron emission current.

3. Instantons in 2-dimensional scalar field theory with a double-well potential

Pure scalar field models do not have instanton solutions. Their presence would automatically mean the possibility of tunneling between classical vacuums.

This, however, is impossible, since it requires infinite energy even at a finite potential barrier. In contrast to similar quantum mechanical tasks in field theory we are talking about the tunneling of one field vacuum state into another in infinite space.

That is, the density of action on the instanton will be finite, and the action will be infinite. To solve the problem, one can consider a field theory model in a two-dimensional space-time in a limited spatial region. We will consider such a region as a circle of length L. That is, geometry $R^1 \otimes S^1$.

So, the Lagrangian of the model has the form:

$$L = \frac{1}{2}\partial_\mu\phi\,\partial_\mu\phi - V(\phi), \qquad \mu = 0,1, \tag{29}$$

where:

$$V(\phi) = \lambda(\phi^2 - \rho^2)^2. \tag{30}$$

Spatially homogeneous solutions in Euclidean space with finite action (instantons) formally coincide with ***static*** kinks of the same model:

$$\phi^{inst}(\tau,x) = \pm\rho\tanh\left((\tau-\tau_0)\rho\sqrt{2\lambda}\right). \tag{31}$$

The most important quantity of interest to us, the action on the instanton, coincides with the energy of the static kink, multiplied by the length of the spatial region:

$$S_E[\phi^{inst}(\tau,x)] = S_0 = \frac{4\sqrt{2\lambda}\rho^3 L}{3}. \tag{32}$$

With exponential accuracy, the tunneling probability amplitude is

$$< -\rho\,|\exp\left(-\hat{H}T\right)|\,\rho > \sim \exp\left(-\frac{4\sqrt{2\lambda}\rho^3 L}{3}\right). \tag{33}$$

Obviously, as the length of the spatial interval goes to infinity, the amplitude goes to zero, as it should be according to the Derrick – Hobart «no-go» theorem [15].

4. Soliton solutions in scalar field theory with a classically degenerate state in space-time (1 + 2)

Any instanton solution in Euclidean space of dimension D is simultaneously a static soliton solution in Minkowski space-time of dimension (1 + D). In this case, the (Euclidean) action on the instanton solution is formally the energy of the corresponding static soliton.

Therefore, the ***static*** soliton solution in space-time $R^1 \otimes S^1 \otimes R^1$ (cylinder geometry) can be immediately written based on the results of Section 3 (see, also, [17]):

$$\phi^{sol}(t,x,y) = \pm\rho \tanh\left(\rho\sqrt{2\lambda}(y-y_0)\right). \tag{34}$$

The *x* coordinate can be considered compactified and the problem can be considered as the simplest model of string theory with a nontrivial classical solution. Such models were used, for example, in the monograph by B. Zwiebach [19]. Along the *x* coordinate, the solutions are homogeneous, which is unimportant when its length is small.

From static solutions, you can go to ***«running»*** ones using the Lorentz transformations:

$$\phi^{sol}(t,x,y) = \pm\rho \tanh\left(\rho\sqrt{2\lambda}\gamma(y-\upsilon t)\right), \tag{35}$$

where υ is the velocity of motion of the moving reference frame relative to the stationary one; γ is the Lorentz factor. Obviously, similar solutions can be obtained for the «sine-Gordon» theory in 3-dimensional space-time.

Conclusion

The instanton method was demonstrated for the first time on the example of the simplest quantum-mechanical tasks on the passing of rectangular and linear potential barriers, which has some methodological importance.

For two-dimensional scalar field-theoretical model in two-dimensional space with a cylinder geometry, instanton solutions are written. It formally coincides with kinks. With the help of Feynman's method of functional integrals, the probability of tunnel transitions is calculated with exponential accuracy. The proposed theory, due to its simplicity, is convenient for modeling the mechanisms of nonperturbative processes of multiple particle production under various energy regimes.

For the same model, but already in three-dimensional space-time, soliton solutions are obtained. It can be used to demonstrate certain effects in string theory. It is important that the smaller the scale of the compactified coordinate, the more significant will be the contribution of classical soliton solutions to the probabilities of various processes described by such theory.

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